

Section 10.4

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Calculus II: August 2016 - December 2016

Problem 5

$$r^2 = \sin(2\theta)$$

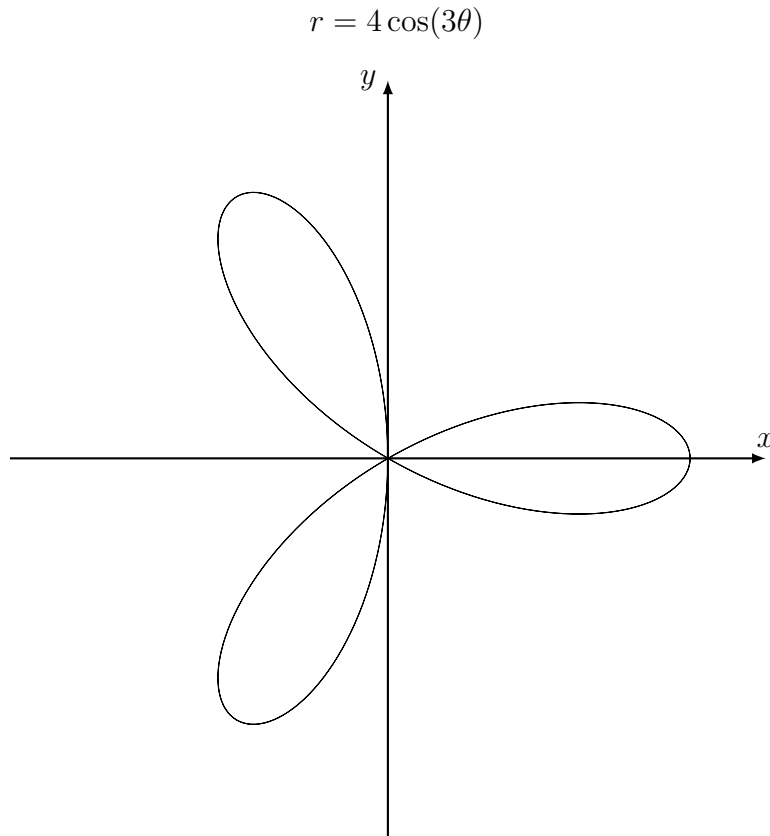
$$\begin{aligned} A &= \frac{1}{2} \int_a^b r^2 d\theta \\ &= \frac{1}{2} \int_0^{\pi/2} \sin(2\theta) d\theta \\ &= \frac{1}{2} \left[\frac{-\cos(2\theta)}{2} \right]_0^{\pi/2} \\ &= \frac{1}{2} \end{aligned}$$

Problem 7

$$r = 4 + 3 \sin(\theta)$$

$$\begin{aligned} A &= \frac{1}{2} \int_a^b r^2 d\theta \\ &= \frac{1}{2} \int_{-\pi/2}^{\pi/2} (4 + 3 \sin(\theta))^2 d\theta \\ &= \frac{1}{2} \int_{-\pi/2}^{\pi/2} 9 \sin^2(\theta) + 24 \sin(\theta) + 16 d\theta \\ &= \frac{1}{2} \int_{-\pi/2}^{\pi/2} \frac{9(1 - \cos(2\theta))}{2} + 24 \sin(\theta) + 16 d\theta \\ &= \frac{1}{2} \int_{-\pi/2}^{\pi/2} \frac{-\cos(2\theta)}{2} + 24 \sin(\theta) + \frac{41}{2} d\theta \\ &= \frac{1}{2} \left[\frac{-9 \sin(2\theta)}{4} - 24 \cos(\theta) + \frac{41}{2} \theta \right]_{-\pi/2}^{\pi/2} = \frac{41\pi}{2} \end{aligned}$$

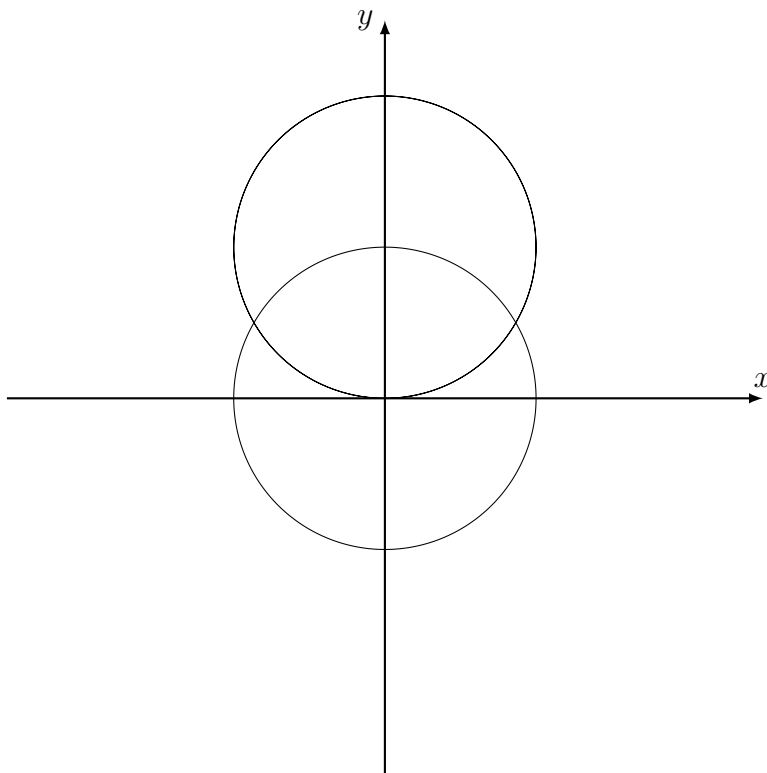
Problem 17



$$\begin{aligned} A &= \frac{1}{2} \int_a^b r^2 d\theta \\ &= \frac{1}{2} \int_{-\pi/6}^{\pi/6} (4 \cos(3\theta))^2 d\theta \\ &= 8 \int_{-\pi/6}^{\pi/6} \cos^2(3\theta) d\theta \\ &= 8 \int_{-\pi/6}^{\pi/6} \frac{1 + \cos(6\theta)}{2} d\theta \\ &= 4 \int_{-\pi/6}^{\pi/6} 1 + \cos(6\theta) d\theta \\ &= 4 \left[\theta + \frac{\sin(6\theta)}{6} \right]_{-\pi/6}^{\pi/6} \\ &= \frac{4\pi}{3} \end{aligned}$$

Problem 23

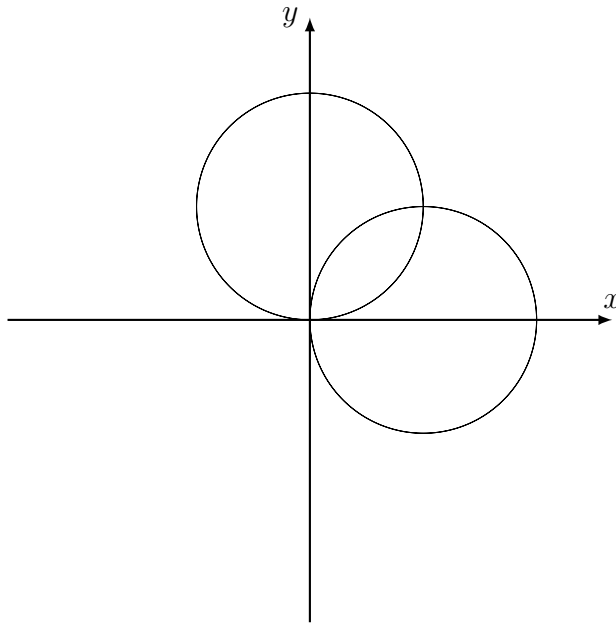
$$r_1 = 2 \quad r_2 = 4 \sin(\theta)$$



$$\begin{aligned}
 A &= \frac{1}{2} \int_a^b (r_2)^2 - (r_1)^2 d\theta \\
 &= \frac{1}{2} \int_{\pi/6}^{5\pi/6} (4 \sin(\theta))^2 - 2^2 d\theta \\
 &= \frac{1}{2} \int_{\pi/6}^{5\pi/6} 16 \sin^2(\theta) - 4 d\theta \\
 &= 2 \int_{\pi/6}^{5\pi/6} 4 \sin^2(\theta) - 1 d\theta \\
 &= 2 \int_{\pi/6}^{5\pi/6} 4 \sin^2(\theta) - 2 + 1 d\theta \\
 &= 2 \int_{\pi/6}^{5\pi/6} 2(2 \sin^2(\theta) - 1) + 1 d\theta \\
 &= 2 \int_{\pi/6}^{5\pi/6} -2 \cos(2\theta) + 1 d\theta \\
 &= 2 \left[-\sin(2\theta) + \theta \right]_{\pi/6}^{5\pi/6} \\
 &= 2 \left(\left[-\sin\left(\frac{5\pi}{3}\right) + \frac{5\pi}{6} \right] - \left[-\sin\left(\frac{\pi}{3}\right) + \frac{\pi}{6} \right] \right) \\
 &= 2\sqrt{3} + \frac{4\pi}{3}
 \end{aligned}$$

Problem 29

$$r = 3 \cos(\theta) \quad r = 3 \sin(\theta)$$



$$\begin{aligned} A &= \frac{1}{2} \int_a^b r^2 d\theta \\ &= \frac{1}{2} (2) \int_0^{\pi/4} (3 \sin(\theta))^2 d\theta \\ &= \int_0^{\pi/4} 9 \sin^2(\theta) d\theta \\ &= \int_0^{\pi/4} 9 \left(\frac{1 - \cos(2\theta)}{2} \right) d\theta \\ &= \frac{9}{2} \int_0^{\pi/4} 1 - \cos(2\theta) d\theta \\ &= \frac{9}{2} \left[\theta - \frac{\sin(2\theta)}{2} \right]_0^{\pi/4} \\ &= \frac{9}{2} \left(\left[\frac{\pi}{4} - \frac{\sin(\frac{\pi}{2})}{2} \right] - \left[0 - \frac{\sin(0)}{2} \right] \right) \\ &= \frac{9\pi}{8} - \frac{9}{4} \end{aligned}$$

Problem 45

$$r = 2 \cos(\theta) \quad 0 \leq \theta \leq \pi$$

$$\begin{aligned}
\frac{dr}{d\theta} &= -2 \sin(\theta) \\
L &= \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \\
&= \int_0^\pi \sqrt{(2 \cos(\theta))^2 + (-2 \sin(\theta))^2} d\theta \\
&= \int_0^\pi \sqrt{4 \cos^2(\theta) + 4 \sin^2(\theta)} d\theta \\
&= \int_0^\pi \sqrt{4(\cos^2(\theta) + \sin^2(\theta))} d\theta \\
&= \int_0^\pi \sqrt{4} d\theta \\
&= \int_0^\pi 2 d\theta \\
&= \left[2\theta \right]_0^\pi \\
&= 2\pi
\end{aligned}$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com