

Section 7.8

Alvin Lin

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Problem 5

$$\begin{aligned} & \int_3^{\infty} \frac{1}{(x-2)^{3/2}} dx \\ & \lim_{b \rightarrow \infty} \int_3^b (x-2)^{\frac{3}{2}} dx \\ & \text{Let : } u = x - 2 \\ & du = dx \\ & \lim_{b \rightarrow \infty} \int_3^b u^{\frac{3}{2}} du \\ & \lim_{b \rightarrow \infty} \left[\frac{-1}{2\sqrt{u}} \right]_3^b \\ & \lim_{b \rightarrow \infty} \left[\frac{-1}{2\sqrt{b}} - \frac{-1}{2\sqrt{3}} \right] \\ & = \frac{\sqrt{3}}{6} \end{aligned}$$

Problem 7

$$\begin{aligned} & \int_{-\infty}^0 \frac{1}{3-4x} dx \\ & \lim_{b \rightarrow -\infty} \int_b^0 \frac{1}{3-4x} dx \\ & \text{Let : } u = 3 - 4x \\ & du = -4dx \\ & \frac{-1}{4} \lim_{b \rightarrow -\infty} \int_b^0 \frac{1}{u} du \\ & \frac{-1}{4} \lim_{b \rightarrow -\infty} \left[\ln |3 - 4x| \right]_b^0 \end{aligned}$$

$$\begin{aligned} \frac{-1}{4} \lim_{b \rightarrow -\infty} \left[\ln |3| - \ln |3 - 4b| \right] \\ = \infty \end{aligned}$$

Problem 13

$$\begin{aligned} \int_{-\infty}^{\infty} x e^{-x^2} dx \\ \int_{-\infty}^0 x e^{-x^2} dx + \int_0^{\infty} x e^{-x^2} dx \end{aligned}$$

Solve as an indefinite integral:

$$\begin{aligned} \int x e^{-x^2} dx \\ \frac{-1}{2} \int -2x e^{-x^2} dx \\ e^{-x^2} + C \end{aligned}$$

Substitute the original limits:

$$\begin{aligned} -\frac{1}{2} \lim_{a \rightarrow -\infty} \left[e^{-x^2} \right]_a^0 - \frac{1}{2} \lim_{b \rightarrow \infty} \left[e^{-x^2} \right]_0^b \\ -\frac{1}{2} \lim_{a \rightarrow -\infty} \left[e^{-a^2} - e^0 \right] - \frac{1}{2} \lim_{b \rightarrow \infty} \left[e^0 - e^{-b^2} \right] \\ = 0 \end{aligned}$$

Problem 29

$$\int_{-2}^{14} (x+2)^{-1/4} dx$$

Solve as an indefinite integral:

$$\int (x+2)^{-1/4} dx$$

$$\text{Let : } f'(x) = x^{-1/4} dx \quad g(x) = x + 2$$

$$f(x) = \frac{4}{3} x^{3/4} \quad g'(x) = dx$$

$$f(g(x)) = \frac{4}{3} (x+2)^{3/4}$$

Substitute the original limits:

$$\frac{4}{3} \left[(x+2)^{3/4} \right]_{-2}^{14}$$

$$\begin{aligned} & \frac{4}{3} \left[(14 + 2)^{3/4} - (-2 + 2)^{3/4} \right] \\ & \frac{4}{3} \left[(14 + 2)^{3/4} - (-2 + 2)^{3/4} \right] \\ & \quad \frac{4}{3} \left[8 - 0 \right] \\ & \quad = \frac{32}{3} \end{aligned}$$

Problem 33

$$\int_0^9 \frac{1}{\sqrt[3]{x-1}} dx$$

Solve as an indefinite integral:

$$\int \frac{1}{\sqrt[3]{x-1}} dx$$

$$\text{Let : } u = x - 1$$

$$du = dx$$

$$\int u^{-2/3} du$$

$$3u^{1/3} + C$$

$$3(x-1)^{1/3} + C$$

Substitute the original limits:

$$3 \left[(x-1)^{1/3} \right]_0^9$$

$$3 \left[\sqrt[3]{0} - \sqrt[3]{8} \right]$$

$$= -6$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com