Section 7.4

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Problem 7

$$\int \frac{x^4}{x-1}\mathrm{d}x$$

$$\begin{array}{r l}
 & x^3 + x^2 + x + 1 \\ \cline{2-2}
x-1) & x^4 \\
 & -x^4 + x^3 \\ \cline{2-2}
 & x^3 \\
 & -x^3 + x^2 \\ \cline{2-2}
 & x^2 \\
 & -x^2 + x \\ \cline{2-2}
 & x \\
 & -x + 1 \\ \cline{2-2}
 & 1
\end{array}$$

$$\int \frac{x^4}{x-1}\mathrm{d}x = \int x^3 + x^2 + x + 1 + \frac{1}{x-1}\mathrm{d}x$$

$$= \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x+1| + C$$

Problem 11

$$\int_0^1 \frac{2}{2x^2+3x+1}\mathrm{d}x$$

$$\frac{2}{2x^2+3x+1} = \frac{2}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}$$

$$(2x+1)(x+1)\frac{2}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}(2x+1)(x+1)$$

$$2 = A(x+1) + B(2x+1)$$
$$2 = Ax + A + 2Bx + B = (A+2B)x + (A+B)$$
$$A + 2B = 0 \quad A + B = 2$$
$$A + 2(2-A) = A + 4 - 2A = 0$$
$$A = 4 \quad B = -2$$
$$\frac{2}{2x^2+3x+1} = \frac{4}{2x+1} - \frac{2}{x+1}$$

Solve it as an indefinite integral:

$$\int \frac{2}{2x^2+3x+1}\mathrm{d}x = \int \frac{4}{2x+1} - \frac{2}{x+1}\mathrm{d}x$$
$$2\int \frac{2}{2x+1}\mathrm{d}x - 2\int \frac{1}{x+1}\mathrm{d}x$$
$$2\int \frac{2}{2x+1}\mathrm{d}x - 2\int \frac{1}{x+1}\mathrm{d}x$$
$$2\ln|2x+1| - 2\ln|x+1| + C$$

Now we apply the original limits of integration:

$$\Big[2\ln|2x+1| - 2\ln|x+1|\Big]_0^1$$
$$2\ln|2(1)+1| - 2\ln|1+1| - (2\ln|2(0)+1| - 2\ln|0+1|)$$
$$2\ln|3| - 2\ln|2| - (0)$$
$$= \ln|\frac{9}{4}|$$

Problem 19

$$\int_0^1 \frac{x^2+x+1}{(x+1)^2(x+2)}\mathrm{d}x$$
$$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{Ax+B}{(x+1)^2} + \frac{D}{x+2}$$
$$(x+1)^2(x+2)\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{Ax+B}{(x+1)^2} + \frac{D}{x+2}(x+1)^2(x+2)$$
$$x^2 + x + 1 = (Ax+B)(x+2) + (D)(x^2+2x+1)$$
$$x^2 + x + 1 = Ax^2 + 2Ax + Bx + 2B + Dx^2 + 2Dx + D$$
$$x^2 + x + 1 = (A+D)x^2 + (2A+B+2D)x + (2B+D)$$
$$A + D = 1 \quad 2A + B + 2D = 1 \quad 2B + D = 1$$

$$2(1-D)+\frac{1-D}{2}+2D=1$$
$$4-4D+1-D+4D=2$$
$$A=-2 \quad B=-1 \quad D=3$$
$$\frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{-2x-1}{(x+1)^2}+\frac{3}{x+2}$$

Solve it as an indefinite integral:

$$\int \frac{-2x-1}{(x+1)^2}+\frac{3}{x+2}\mathrm{d}x$$

$$-\int \frac{2x+1}{(x+1)^2}\mathrm{d}x+\int \frac{3}{x+2}\mathrm{d}x$$

$$3\ln|x+2|-\int \frac{2x+1}{(x+1)^2}\mathrm{d}x$$

$$Let: u=x+1 \quad x=u-1 \quad \mathrm{d}u=\mathrm{d}x$$

$$3\ln|x+2|-\int \frac{2(u-1)+1}{u^2}\mathrm{d}u$$

$$3\ln|x+2|-\int \frac{2u-1}{u^2}\mathrm{d}u$$

$$3\ln|x+2|-\int \frac{2}{u}\mathrm{d}u-\int u^{-2}\mathrm{d}u$$

$$3\ln|x+2|-2\ln|u|+\frac{1}{u}+C$$

$$3\ln|x+2|-2\ln|x+1|+\frac{1}{x+1}+C$$

Now we apply the original limits of integration:

$$\left[3\ln|x+2|-2\ln|x+1|+\frac{1}{x+1}\right]_0^1$$

$$3\ln|1+2|-2\ln|1+1|+\frac{1}{1+1}-(3\ln|0+2|-2\ln|0+1|+\frac{1}{0+1})$$

$$3\ln|3|-2\ln|2|+\frac{1}{2}-(3\ln|2|-0+1)$$

$$3\ln|3|-2\ln|2|-3\ln|2|-\frac{1}{2}$$

Problem 23

$$\int \frac{10}{(x-1)(x^2+9)}$$

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+D}{x^2+9}$$

$$(x-1)(x^2+9)\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+D}{x^2+9}(x-1)(x^2+9)$$

$$10 = A(x^2+9) + (Bx+D)(x-1)$$

$$10 = Ax^2 + 9A + Bx^2 - Bx + Dx - D$$

$$10 = (A+B)x^2 + (D-B)x + (9A-D)$$

$$A+B=0 \quad D-B=0 \quad 9A-D=10$$

$$D = 9A-10 \quad 9A-10-B=0 \quad 9A-10-(-A)=10$$

$$10A = 20$$

$$A=2 \quad B=-2 \quad D=8$$

$$\frac{10}{(x-1)(x^2+9)} = \frac{2}{x-1} + \frac{-2x+8}{x^2+9}$$

$$\int \frac{10}{(x-1)(x^2+9)}\mathrm{d}x = \int \frac{2}{x-1}\mathrm{d}x - \int \frac{2x-8}{x^2+9}\mathrm{d}x$$

$$2\int \frac{1}{x-1}\mathrm{d}x - \int \frac{2x}{x^2+9}\mathrm{d}x + \int \frac{8}{x^2+9}\mathrm{d}x$$

$$2\ln|x-1| - \ln|x^2+9| + \frac{8}{3}\int \frac{3}{x^2+9}\mathrm{d}x$$

$$2\ln|x-1| - \ln|x^2+9| + \frac{8}{3}\tan^{-1}(\frac{x}{3}) + C$$

Problem 39

$$\int \frac{\sqrt{x+1}}{x}\mathrm{d}x$$

$$Let: u = \sqrt{x+1}$$

$$x = u^2 - 1$$

$$\mathrm{d}u = \frac{1}{2\sqrt{x+1}}\mathrm{d}x$$

$$2\mathrm{d}u\sqrt{x+1} = \mathrm{d}x$$

$$2u\mathrm{d}u = \mathrm{d}x$$

$$\int \frac{u}{u^2-1} 2u \mathrm{d}u$$

$$2\int \frac{u^2}{u^2-1} \mathrm{d}u$$

$$2\int \frac{u^2-1+1}{u^2-1} \mathrm{d}u$$

$$2\int \frac{u^2-1}{u^2-1} + \frac{1}{u^2-1} \mathrm{d}u$$

$$2\int 1 \mathrm{d}u + 2\int \frac{1}{(u-1)(u+1)} \mathrm{d}u$$

$$2u + C + 2\int \frac{1}{(u-1)(u+1)} \mathrm{d}u$$

$$\frac{1}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}$$

$$(u-1)(u+1)\frac{1}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}(u-1)(u+1)$$

$$1 = A(u+1) + B(u-1)$$

$$1 = Au + A + Bu - B$$

$$1 = (A+B)u + (A-B)$$

$$A + B = 0 \quad A - B = 1$$

$$(1+B) + B = 0 \quad B = -\frac{1}{2} \quad A = \frac{1}{2}$$

$$\frac{1}{(u-1)(u+1)} = \frac{1}{2(u-1)} - \frac{1}{2(u+1)}$$

$$\int \frac{1}{(u-1)(u+1)} \mathrm{d}u = \int \frac{1}{2(u-1)} - \frac{1}{2(u+1)} \mathrm{d}u$$

$$\frac{1}{2}\int \frac{1}{u-1} - \frac{1}{u+1} \mathrm{d}u$$

$$\frac{1}{2}(\ln|u-1| - \ln|u+1|) + C$$

$$2u + C + 2\int \frac{1}{(u-1)(u+1)} \mathrm{d}u = 2u + 2(\frac{1}{2}(\ln|u-1| - \ln|u+1|)) + C$$

$$2u + \ln|u-1| - \ln|u+1| + C$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com