

Section 7.2

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Problem 1

$$\begin{aligned} & \int \sin^2(x) \cos^3(x) dx \\ & \int \sin^2(x) \cos^2(x) \cos(x) dx \\ & \int \sin^2(x) (1 - \sin^2(x)) \cos(x) dx \\ & \text{Let : } u = \sin(x) \\ & du = \cos(x) dx \\ & \int u^2 (1 - u^2) du \\ & \int u^2 - u^4 du \\ & \frac{1}{3} u^3 - \frac{1}{5} u^5 + C \\ & \frac{1}{3} \sin^3(x) - \frac{1}{5} \sin^5(x) + C \end{aligned}$$

Problem 7

$$\begin{aligned} & \int_0^{\pi/2} \cos^2(\theta) d\theta \\ & \int_0^{\pi/2} \frac{\cos(2\theta) + 1}{2} d\theta \\ & \frac{1}{2} \int_0^{\pi/2} \cos(2\theta) + 1 d\theta \\ & \frac{1}{2} \left[\frac{1}{2} \sin(2\theta) + \theta \right]_0^{\pi/2} \\ & \frac{1}{2} \left[0 + \frac{\pi}{2} - (0 + 0) \right] \\ & = \frac{\pi}{4} \end{aligned}$$

Problem 27

$$\int \tan^3(x) \sec(x) dx$$
$$\int \tan^2(x) \tan(x) \sec(x) dx$$

Knowing this, we want to isolate a $\tan^2(x)$ because $\tan^2(x) = \sec^2(x) - 1$

$$\int (\sec^2(x) - 1) \tan(x) \sec(x) dx$$

$$\text{Let : } u = \sec(x)$$

$$du = \sec(x) \tan(x) dx$$

$$\int u^2 - 1 du$$

$$\frac{1}{3} u^3 - u + C$$

$$\frac{1}{3} \sec^3(x) - \sec(x) + C$$

Problem 37

$$\int_{\pi/4}^{\pi/2} \cot^5(\phi) \csc^3(\phi) d\phi$$
$$\int_{\pi/4}^{\pi/2} \cot^4(\phi) \csc^2(\phi) \cot(\phi) \csc(\phi) d\phi$$
$$\int_{\pi/4}^{\pi/2} \cot^4(\phi) \csc^2(\phi) \cot(\phi) \csc(\phi) d\phi$$
$$\int_{\pi/4}^{\pi/2} (\csc^2(\phi) - 1)^2 \csc^2(\phi) \cot(\phi) \csc(\phi) d\phi$$

For now we can solve it as an indefinite integral using u -substitution.

$$\text{Let : } u = \csc(\phi)$$

$$du = -\csc(\phi) \cot(\phi) d\phi$$

$$- \int (u^2 - 1)^2 u^2 du$$

$$- \int (u^2 - 1)^2 u^2 du$$

$$- \int (u^4 - 2u^2 + 1) u^2 du$$

$$- \int u^6 - 2u^4 + u^2 du$$

$$- \left[\frac{1}{7} u^7 - \frac{2}{5} u^5 + \frac{1}{3} u^3 \right]$$

$$\left[-\frac{1}{5} \csc^7(\phi) + \frac{2}{5} \csc^5(\phi) - \frac{1}{3} \csc^3(\phi) \right]_{\pi/4}^{\pi/2}$$

Problem 41

$$\begin{aligned} & \int \sin(8x) \cos(5x) dx \\ & \int \frac{1}{2} [\sin(8x - 5x) + \sin(8x + 5x)] dx \\ & \frac{1}{2} \left[\int \sin(3x) dx + \int \sin(13x) dx \right] \\ & \frac{1}{6} \int 3 \sin(3x) dx + \frac{1}{26} \int 13 \sin(13x) dx \\ & -\frac{\cos(3x)}{6} - \frac{\cos(13x)}{26} + C \end{aligned}$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com