Section 7.1

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Section 5.5

Problem 3

$$\int x\cos(5x)\mathrm{d}x$$

$$Let: u = x \quad \mathrm{d}v = \cos(5x)\mathrm{d}x$$

$$\mathrm{d}u = \mathrm{d}x \quad v = \frac{\sin(5x)}{5}$$

$$\int x\cos(5x)\mathrm{d}x = \frac{x\sin(5x)}{5} - \frac{1}{5}\int \sin(5x)\mathrm{d}x$$

$$= \frac{x\sin(5x)}{5} - \frac{1}{5}(-\frac{\cos(5x)}{5}) + C$$

$$= \frac{1}{5}x\sin(5x) + \frac{1}{25}\cos(5x) + C$$

Problem 11

$$\int t^4\ln(t)\mathrm{d}t$$

$$Let: u = \ln(t) \quad \mathrm{d}(v) = t^4\mathrm{d}t$$

$$\mathrm{d}u = \frac{1}{t}\mathrm{d}t \quad v = \frac{t^5}{5}$$

$$\int t^4\ln(t)\mathrm{d}t = \frac{t^5\ln t}{5} - \int \frac{t^5}{5t}\mathrm{d}t$$

$$= \frac{1}{5}t^4\ln(t) - \frac{1}{5}\int t^4\mathrm{d}t = \frac{1}{5}t^4\ln(t) - \frac{1}{5}\left[\frac{t^5}{5}\right]$$

$$= \frac{1}{5}t^4\left(\ln(t) - \frac{t}{5}\right)$$

Problem 17

$$\int e^{2\theta}\sin(3\theta)d(\theta)$$

$$Let: u=\sin(3\theta) \quad dv=e^{2\theta}d\theta$$

$$du=3\cos(3\theta)d\theta \quad v=\frac{e^{2\theta}}{2}$$

$$\int e^{2\theta}\sin(3\theta)d(\theta)=\frac{e^{2\theta}\sin(3\theta)}{2}-\int\frac{3e^{2\theta}\cos(3\theta)}{2}d\theta$$

$$\int e^{2\theta}\sin(3\theta)d(\theta)=\frac{e^{2\theta}\sin(3\theta)}{2}-\frac{3}{2}\int e^{2\theta}\cos(3\theta)$$

We must use integration by parts again to separate the integral.

$$\int e^{2\theta}\cos(3\theta)$$

$$Let: u=\cos(3\theta) \quad dv=e^{2\theta}d\theta$$

$$du=-3\sin(3\theta)d\theta \quad v=\frac{e^{2\theta}}{2}$$

$$\int e^{2\theta}\cos(3\theta)=\frac{e^{2\theta}\cos(3\theta)}{2}-\int-\frac{3}{2}e^{2\theta}\sin(3\theta)d\theta$$

$$=\frac{e^{2\theta}\cos(3\theta)}{2}+\frac{3}{2}\int e^{2\theta}\sin(3\theta)d\theta$$

We can form the following equation from this:

$$\int e^{2\theta}\sin(3\theta)d\theta=\frac{e^{2\theta}\sin(3\theta)}{2}-\frac{3}{2}\Big(\frac{e^{2\theta}\cos(3\theta)}{2}+\frac{3}{2}\int e^{2\theta}\sin(3\theta)d\theta\Big)$$

$$\int e^{2\theta}\sin(3\theta)d\theta=\frac{e^{2\theta}\sin(3\theta)}{2}-\frac{3e^{2\theta}\cos(3\theta)}{4}-\frac{9}{4}\int e^{2\theta}\sin(3\theta)d\theta\Big)$$

$$\frac{13}{4}\int e^{2\theta}\sin(3\theta)d\theta=\frac{e^{2\theta}\sin(3\theta)}{2}-\frac{3e^{2\theta}\cos(3\theta)}{4}$$

$$=\int e^{2\theta}\sin(3\theta)d\theta=\frac{2e^{2\theta}\sin(3\theta)}{13}-\frac{3e^{2\theta}\cos(3\theta)}{13}$$

Problem 19

$$\int z^3e^z dz$$

$$Let: u=z^3 \quad dv=e^z dz$$

$$du=3z^2dz \quad v=e^z$$

$$\int z^3 e^z dz = z^3 e^z - \int 3z^2 e^z = z^3 e^z - 3\int z^2 e^z$$

We can use integration by parts again to reduce the new integral.

$$\int z^2 e^z$$

$$\textit{Let} : u = z^2 \quad dv = e^z dz$$

$$du = 2z dz \quad v = e^z$$

$$\int z^2 e^z = z^2 e^z - \int 2z e^z dz = z^2 e^z - 2\int z e^z dz$$

We can use integration by parts one more time to remove the z from the integral.

$$\int z e^z dz$$

$$\textit{Let} : u = z \quad dv = e^z dz$$

$$du = dz \quad v = e^z$$

$$\int z e^z dz = z e^z - \int e^z dz$$

$$= z e^z - e^z$$

Now we put all this shit back together.

$$\int z^3 e^z dz = z^3 e^z - 3\int z^2 e^z$$

$$z^3 e^z - 3\left(z^2 e^z - 2\int z e^z dz\right)$$

$$z^3 e^z - 3z^2 e^z + 6\int z e^z dz$$

$$z^3 e^z - 3z^2 e^z + 6(z e^z - e^z)$$

$$= z^3 e^z - 3z^2 e^z + 6z e^z - 6e^z$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com