

Section 6.3

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Calculus II: August 2016 - December 2016

Problem 1

$$f(x) = x(x - 1)^2$$

$$V = 2\pi \int_a^b x f(x) dx$$

$$V = 2\pi \int_0^1 x(x(x - 1)^2) dx$$

$$V = 2\pi \int_0^1 x^2(x^2 - 2x + 1) dx$$

$$V = 2\pi \int_0^1 x^4 - 2x^3 + x^2 dx$$

$$V = 2\pi \left[\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} \right]_0^1$$

$$V = \frac{\pi}{15}$$

Problem 7

$$f(x) = (6x - 2x^2) - x^2 = 6x - 3x^2$$

$$V = 2\pi \int_a^b x f(x) dx$$

$$V = 2\pi \int_0^2 x(6x - 3x^2) dx$$

$$V = 2\pi \int_0^2 6x^2 - 3x^3 dx$$

$$V = 2\pi \left[\frac{6x^3}{3} - \frac{3x^4}{4} \right]_0^2$$

$$V = 8\pi$$

Problem 15

$$y = x^3$$

bounded by $x = 0$, $y = 8$, rotated about $x = 3$.

$$f(x) = 8 - x^3 \quad r = 3 - x$$

$$V = 2\pi \int_a^b r f(x) dx$$

$$V = 2\pi \int_0^2 (3 - x)(8 - x^3) dx$$

$$V = 2\pi \int_0^2 x^4 - 3x^3 - 8x + 24 dx$$

$$V = 2\pi \left[\frac{x^5}{5} - \frac{3x^4}{4} - \frac{8x^2}{2} + 24x \right]_0^2$$

$$V = \frac{264\pi}{5}$$

Problem 21a

$$f(x) = xe^{-x}$$

$$V = 2\pi \int_a^b x f(x) dx$$

$$V = 2\pi \int_0^2 x(xe^{-x}) dx$$

$$V = 2\pi \int_0^2 x^2 e^{-x} dx$$

Problem 41

$$x^2 + (y - 1)^2 = 1$$

$$f(x) = (1 + \sqrt{1 - x^2}) - (1 - \sqrt{1 - x^2}) = 2\sqrt{1 - x^2}$$

$$V = 2\pi \int_a^b x f(x) dx$$

$$V = 2\pi \int_0^1 x(2\sqrt{1 - x^2}) dx$$

$$V = 4\pi \int_0^1 x\sqrt{1 - x^2} dx$$

$$\text{Let : } u = 1 - x^2$$

$$du = -2x dx$$

$$V = 4\pi \int \sqrt{u} du$$

$$V = 4\pi \left[\frac{2}{3} (1 - x^2)^{\frac{3}{2}} \right]_0^1$$

$$V = \frac{4\pi}{3}$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com