

Section 5.5

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Problem 7

$$\begin{aligned} & \int x\sqrt{1-x^2}dx \\ \text{Let : } & u = 1 - x^2 \\ & du = -2xdx \\ & \int x\sqrt{u} \frac{du}{-2x} \\ & -\frac{1}{2} \int \sqrt{u}du \\ & -\frac{1}{2} \left(\frac{2}{3} u^{\frac{3}{2}} \right) \\ & = -\frac{1}{3} (1-x^2)^{\frac{3}{2}} + C \end{aligned}$$

Problem 15

$$\begin{aligned} & \int \cos^3(\theta) \sin(\theta) d\theta \\ \text{Let : } & u = \cos(\theta) \\ & du = -\sin(\theta) d\theta \\ & \int u^3 \sin(\theta) \frac{du}{-\sin(\theta)} \\ & - \int u^3 du \\ & - \left(\frac{u^4}{4} \right) \\ & = -\frac{\cos^4 \theta}{4} + C \end{aligned}$$

Problem 21

$$\int \frac{(\ln x)^3}{x} dx$$

$$\text{Let : } u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \frac{u^3}{x} dx$$

$$\int u^3 dx$$

$$\left(\frac{u^4}{4}\right)$$

$$= \frac{(\ln x)^4}{4} + C$$

Problem 25

$$\int e^x \sqrt{1 + e^x} dx$$

$$\text{Let : } u = 1 + e^x$$

$$du = e^x dx$$

$$\int e^x \sqrt{u} \frac{du}{e^x}$$

$$\int \sqrt{u} du$$

$$\frac{2}{3} u^{\frac{3}{2}}$$

$$= \frac{2}{3} (1 + e^x)^{\frac{3}{2}} + C$$

Problem 45

$$\int \frac{1+x}{1+x^2} dx$$

$$\text{Let : } u = 1 + x^2$$

$$du = 2x dx$$

$$\int \frac{1+x}{1+x^2} = \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$$

$$\int \frac{1}{1+x^2} dx + \int \frac{x du}{u 2x}$$

$$\int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{1}{u} du$$

$$\tan^{-1}(x) + \frac{1}{2} \ln u$$

$$\tan^{-1}(x) + \frac{1}{2} \ln(1+x^2) + C$$

Problem 67

$$\int_1^2 x\sqrt{x-1} dx$$

$$\text{Let : } u = x - 1$$

$$du = dx$$

The integral from 1 to 2 becomes 0 to 1 in terms of u .

$$\int_0^1 (u+1)\sqrt{u} du$$

$$\int_0^1 u^{\frac{3}{2}} + u^{\frac{1}{2}} du$$

$$\left[\frac{2}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}} \right]_0^1$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{16}{15}$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com