

Average Value of a Function

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Average Value of a Function

Recall the notion of averaging. Given:

$$x_1, x_2, x_3$$
$$x_{average} = \frac{x_1 + x_2 + x_3}{3}$$

The average value of a function from $x = a$ to $x = b$ is:

$$f_{av} = \frac{1}{b-a} \int_a^b f(x) dx$$

This is a continuous version of the standard average formula, which uses discrete points instead.

The Mean Value Theorem of Integrals

If f is continuous on $[a, b]$, then there exists c in $[a, b]$ such that

$$f(c) = f_{av} = \frac{1}{b-a} \int_a^b f(x) dx$$

Therefore:

$$\int_a^b f(x) dx = (b-a)f(c)$$

Practice Problem 9

$$f(x) = (x - 3)^2 \quad [2, 5]$$

$$f_{av} = \frac{1}{b - a} \int_a^b f(x) dx$$

$$\frac{1}{5 - 2} \int_2^5 (x - 3)^2 dx$$

$$\frac{1}{3} \left[\frac{(x - 3)^3}{3} \right]_2^5$$

$$\frac{1}{9} \left[(x - 3)^3 \right]_2^5$$

$$= \frac{1}{9} (8 + 1) = 1$$

$$f(c) = f_{av} = 1$$

$$(c - 3)^3 = 1$$

$$c^2 - 6c + 9 = 1$$

$$c^2 - 6c + 8 = 1$$

$$(c - 2)(c - 4) = 1$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com