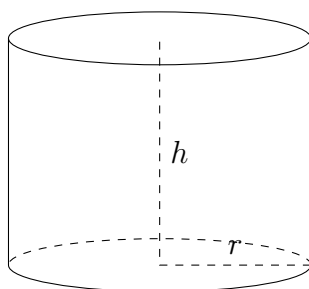


# Volumes By Integration (Shells)

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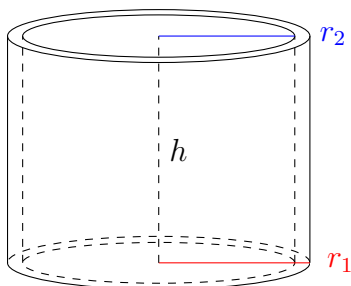
Calculus II: August 2016 - December 2016

## Volumes By Integration (Shells)



With the disk method of volume by integration, you can imagine this problem as an integration problem where a cross section of the cylinder is rotated an axis.

There is another method that uses infinitely small shells which compose the volume.



The volume of the disk is:

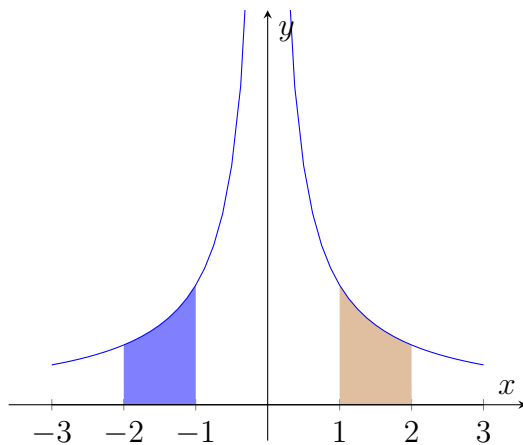
$$V = \pi((r_1)^2 - (r_2)^2)h$$
$$V = \lim_{r_1 \rightarrow r_2} 2\pi \frac{r_1 + r_2}{2} (r_1 - r_2)h$$
$$V = 2\pi \bar{r}(\Delta r)h$$

And as a general form, the sums of the volumes of all the disks that compose the figure is:

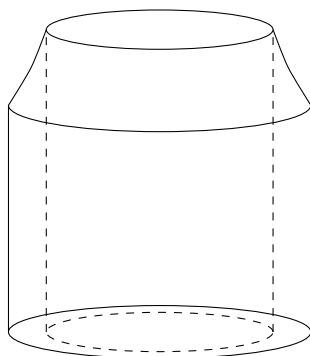
$$V = \int_a^b 2\pi x f(x) dx$$

### Example 1

$$y = \frac{1}{x} \quad y = 0 \quad x = 1 \quad x = 2$$



When the red section is rotated about the y-axis, it passes through the highlighted blue section and the following shape results:



$$V = \int_1^2 2\pi x f(x) dx$$

$$V = 2\pi \int_1^2 x \frac{1}{x} dx$$

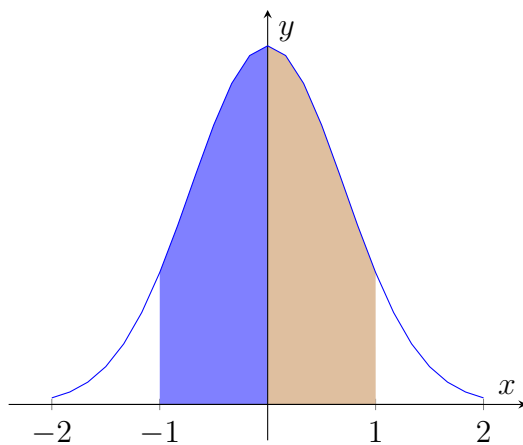
$$V = 2\pi \int_1^2 dx$$

$$V = 2\pi \left[ x \right]_1^2$$

$$V = 2\pi[2 - 1] = 2\pi$$

## Example 2

$$y = e^{-x^2} \quad y = 0 \quad x = 0 \quad x = 1$$



$$V = \int_0^1 2\pi x f(x) dx$$

$$V = \int_0^1 2\pi x e^{-x^2} dx$$

$$V = -\pi \int_0^1 -2x e^{-x^2} dx$$

$$V = \pi \left[ e^{-x^2} \right]_0^1$$

$$V = \pi \left[ e^0 - e^{-1} \right]$$

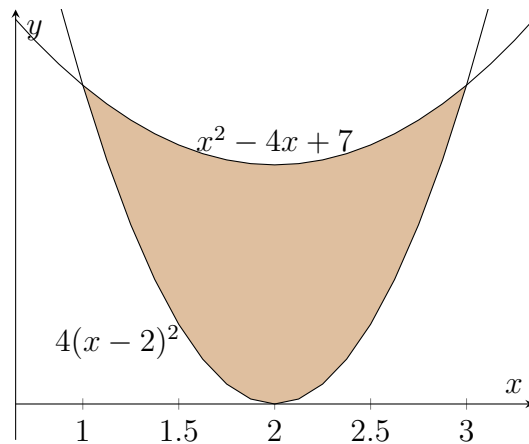
$$V = \pi \left[ 1 - \frac{1}{e} \right]$$

$$V = \pi - \frac{\pi}{e}$$

### Practice Problem 7

$$y = 4(x - 2)^2 \quad y = x^2 - 4x + 2$$

revolved around the y-axis.



$$V = 2\pi \int_1^3 x \left[ (x^2 - 4x + 7) + (4(x - 2)^2) \right] dx$$

$$V = 2\pi \int_1^3 x \left[ x^3 - 4x + 7 - 4x^2 - 16 + 16x \right] dx$$

$$V = 2\pi \int_1^3 -3x^3 + 12x^2 - 9x dx$$

$$V = 2\pi \left[ \frac{-3x^4}{4} + \frac{12x^3}{3} - \frac{9x^2}{2} \right]_1^3$$

$$V = 16\pi$$

If any errors are found, please contact me at [alvin.lin.dev@gmail.com](mailto:alvin.lin.dev@gmail.com)