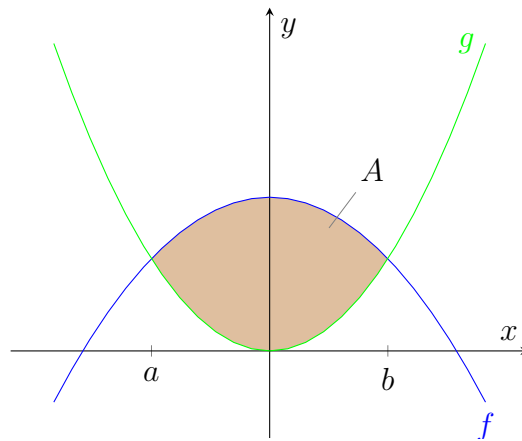


Areas Between Curves

Alvin Lin

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Areas Between Curves



Finding the area between curves is very similar to finding the area underneath a curve. The area between f and g is simply the difference between the area under f and the area under g .

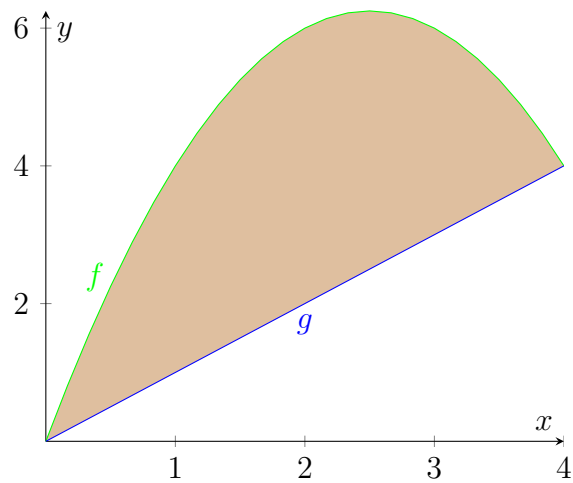
$$A = \int_a^b [f(x) - g(x)] dx$$

Practice Problem 1

Find the area between the curves:

$$y = 5x - x^2$$

$$y = x$$



We need to find the points of intersection between the curves:

$$x = 5x - x^2$$

$$x^2 - 4 = 0$$

$$x(x - 4) = 0$$

$$x = 0 \quad x = 4$$

Now we can use those as the limits of integration:

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_0^4 [5x - x^2 - x] dx$$

$$\int_0^4 [4x - x^2] dx$$

$$\left[\frac{4x^2}{2} - \frac{x^3}{3} \right]_0^4$$

$$\frac{4(4^2)}{2} - \frac{4^3}{3} - (0 - 0)$$

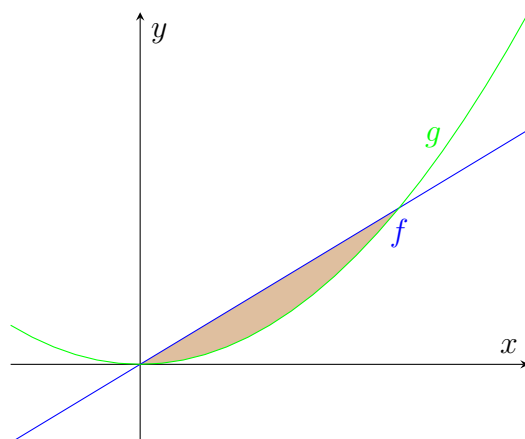
$$= 32 - \frac{64}{3} = \frac{32}{3}$$

Practice Problem 2

Find the area between the curves:

$$y = x$$

$$y = x^2$$



We need to find the points of intersection between the curves:

$$x = x^2$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0 \quad x = 1$$

Now we can use these as the limits of integration:

$$A = \int_0^1 x - x^2 dx$$

$$\left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1$$

$$\frac{1}{2} - \frac{1}{3} - (0 - 0)$$

$$= \frac{1}{6}$$

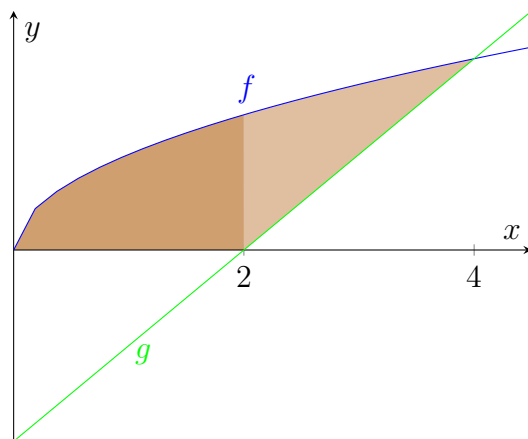
Practice Problem 7

Find the area between the curves in the first quadrant:

$$y = \sqrt{x}$$

$$y = x - 2$$

This problem is trickier, we must split it into two integrals.



$$A = \int_0^2 \sqrt{x} dx + \int_2^4 \sqrt{x} - (x - 2) dx$$

The first integral is just the area under the curve of \sqrt{x} from 0 to 2, while the second integral is the area between the curves from 2 to 4.

$$\begin{aligned} & \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^2 + \left[\frac{2}{3} x^{\frac{3}{2}} - \frac{1}{2} x^2 + 2x \right]_2^4 \\ & \frac{2}{3} (2^{\frac{3}{2}}) + \left[\frac{2}{3} 4^{\frac{3}{2}} - \frac{1}{2} 4^2 + 2(4) \right] - \left[\frac{2}{3} 2^{\frac{3}{2}} - \frac{1}{2} 2^2 + 2(2) \right] \\ & \frac{2}{3} (2^{\frac{3}{2}}) + \frac{2}{3} 4^{\frac{3}{2}} - \frac{1}{2} 4^2 + 2(4) - \frac{2}{3} 2^{\frac{3}{2}} + \frac{1}{2} 2^2 - 2(2) \\ & \frac{16}{3} - 8 + 8 + 4 - 4 \\ & = \frac{16}{3} \end{aligned}$$

If any errors are found, please contact me at alvin.lin.dev@gmail.com