University Physics 2

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Electric Potential

Conservation of Energy:

$$\Delta K + \Delta U = W_{ext}$$

The change in potential energy plus the change in kinetic energy is equal to the external work by non-conservative forces. If there is no outside work:

$$\begin{aligned} \Delta K + \Delta U &= 0 \\ E_i &= E_f \\ K_i + U_i &= K_f + U_f \end{aligned}$$

K is usually $\frac{1}{2}mv^2$ while U depends on the specifics of the problem. Also recall the formula for work:

$$W = \vec{F} \cdot \Delta\vec{r}$$

With many different forces:

$$W = \sum \vec{F} \cdot \Delta\vec{r_i}$$

$\vec{F}$ will generally not be constant, which requires us to integrate:

$$W = \int_i^f \vec{F} \cdot \, \mathrm{d}\vec{r}$$

If a potential energy is doing the work with no external non-conservative force:

$$W = -\Delta U_{internal}$$

For electrical potential, we will start with:

$$W = \int \vec{F} \cdot \, \mathrm{d}r$$

We can assume this is for a charge q in a constant field.

$$W = \int (q\vec{E}) \cdot \, \mathrm{d}\vec{r}$$

Let the force be in the same direction as the electric field.

$$W = qE\Delta x$$

This yields the following relation:

$$\Delta U = -W = -qE\Delta x$$

Just like for Coulomb's Law, we can divide out the test charge:

$$\text{Electric potential} = \Delta V = \frac{\Delta U}{q} = -E\Delta x$$

Using this, we can also derive the formula for potential difference between two points:

$$W = -\Delta U = -q\Delta V = \int_i^f \vec{F} \cdot \, \mathrm{d}r$$

$$\begin{aligned}
\Delta V &= -\frac{1}{q}\int_i^f \vec{F} \cdot \, \mathrm{d}\vec{r} \\
&= -\int_i^f \frac{\vec{F}}{q} \cdot \, \mathrm{d}\vec{r} \\
&= -\int_i^f \vec{E} \cdot \, \mathrm{d}\vec{r} \\
&= V_f - V_i
\end{aligned}$$

Electric potential is measured in volts, where one volt is equal to one joule per Coulomb.

Equipotential Lines

Equipotential lines are like a topographic map. Along an equipotential line, voltage and potential energy are constant (these are scalars, not vectors, $\Delta V = 0$). So along a line:

$$W = -\Delta U = -q\Delta v = 0$$

No work is done, nor does potential energy change:

$$W = \int \vec{F} \cdot \, \mathrm{d}\vec{r} = \int (q\vec{E}) \cdot \, \mathrm{d}\vec{r} = 0$$

For all fields: $\vec{E} \perp \mathrm{d}\vec{r}$. The electric field is perpendicular to the equipotential lines at all points and points from high to low. The region with the largest electric field is where the equipotential lines are closest together.

Voltage Difference for a Point Charge

If we ask for V_r, this means with respect to infinity, where $V(\infty) = 0$.

$$\begin{aligned}
\Delta V = V_\infty - V_r &= -\int_r^\infty \vec{E} \cdot \, \mathrm{d}\vec{r} \\
V_r &= \int_r^\infty \vec{E} \cdot \, \mathrm{d}\vec{r} \\
\mathrm{d}\vec{r} &= \mathrm{d}r' \hat{r} \\
V_r &= \int_r^\infty \frac{kq}{r'^2} \, \mathrm{d}r' \\
&= \left[-kq\frac{1}{r} \right]_r^\infty \\
&= \frac{1}{4\pi\epsilon_\circ} \frac{q}{r} = \frac{kq}{r}
\end{aligned}$$

With multiple point charges:

$$V_{total} = \sum_{point\ charge} = V_1 + V_2 + V_3 + \ldots$$

The potential energy between two point charges is:

$$\Delta U = q\Delta V = \frac{kq_1q_2}{r}$$

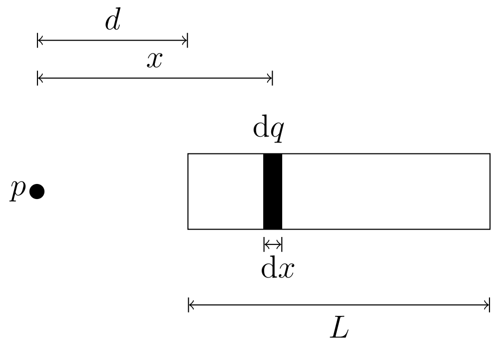
To find $V(r)$ for a charge distribution:

$$\mathrm{d}V = \frac{k \, \mathrm{d}q}{r}$$

This goes through a process similar to finding the electric field given a charge distribution.

Example

Assume a constant charge density of λ:



$$
\begin{aligned}
\mathrm{d}V &= \frac{k\ \mathrm{d}q}{x} \\
&= \frac{k\lambda\ \mathrm{d}x}{x} \\
V &= \int_d^{d+L} \frac{k\lambda\ \mathrm{d}x}{x} \\
&= k\lambda \Big[\ln(x) \Big]_d^{d+L} \\
&= k\lambda \ln\left(\frac{d}{d+L}\right)
\end{aligned}
$$

Unlike the electric field problems, the voltage is a scalar, so we don't need to break it into its components. The voltage is also proportional to $\frac{1}{r}$ instead of $\frac{1}{r^2}$.

Finding the Electric Field from Voltage

We know the following:

$$V_b - V_a = -\int_a^b \vec{E} \cdot\ \mathrm{d}\vec{r}$$

Also by definition:

$$V_b - V_a = \int_a^b\ \mathrm{d}V$$

Therefore:

$$\begin{aligned}
\int_a^b \mathrm{d}V &= -\int_a^b \vec{E} \cdot \mathrm{d}\vec{r} \\
\mathrm{d}V &= -\vec{E} \cdot \mathrm{d}\vec{r} \\
\mathrm{d}r &= \mathrm{d}x\hat{i} + \mathrm{d}y\hat{j} + \mathrm{d}z\hat{k} \\
-\mathrm{d}V &= E_x \, \mathrm{d}x\hat{i} + E_y \, \mathrm{d}y\hat{j} + E_z \, \mathrm{d}z\hat{k} \\
E_x &= -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z - \frac{\partial V}{\partial z} \\
\vec{E} &= E_x\hat{i} + E_y\hat{j} + E_z\hat{k} \\
&= -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) \\
&= -\nabla V
\end{aligned}$$

Thus the electric field is just the gradient of the voltage.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech