

# University Physics 2

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## Electric Flux

Light, magnetism, heat, and water can all have flux. Flux is defined as the flow of some field through a surface.

$$\Phi_E = \vec{E} \cdot \vec{A}$$
$$\vec{A} = A\vec{n}$$

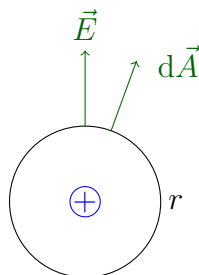
$\vec{A}$  is an area centered on the surface, with the magnitude equal to the area and the direction pointing normal to the surface. For a figure with multiple surfaces:

$$\Phi_E = \sum \vec{E} \cdot \Delta\vec{A}$$

Gauss's Law states that the electric flux through a closed surface is proportional to the charge inside the surface (denoted  $Q_{enc}$ ).

$$\Phi_E = \frac{Q_{enc}}{\epsilon_0} \quad K = \frac{1}{4\pi\epsilon_0}$$
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

## Gauss's Law for a Point Charge



Use Gauss's Law to find  $\vec{E}$  at  $r$  given a point charge surrounded by a spherical shell of radius  $r$ :

$$\begin{aligned} \oint \vec{E} \cdot d\vec{A} &= \frac{Q_{enc}}{\epsilon_0} \\ &= \oint |\vec{E}| |d\vec{A}| \cos \theta \\ \theta &= 0 \text{ via spherical symmetry} \\ &= \oint |\vec{E}| |d\vec{A}| \\ &= |\vec{E}| \oint |d\vec{A}| \text{ since } E \text{ is constant over } r \\ &= |\vec{E}| 4\pi r^2 \\ |\vec{E}| 4\pi r^2 &= \frac{Q}{\epsilon_0} \\ |\vec{E}| &= \frac{1}{4\pi r^2} \frac{Q}{\epsilon_0} \text{ (Coulomb's Law)} \end{aligned}$$

## Solving Gauss's Law

1. Draw a picture with axes, distances, coordinates,  $dq$ ,  $\vec{E}$  fields,  $d\vec{A}$ .
2. Decide how to solve the problem. For 3D symmetric charge distribution, use Gauss's Law. For 2D rings or lines, use the integral form of Coulomb's Law.
3. Pick a Gaussian surface. For objects with spherical symmetry, use a shell. For objects with cylindrical symmetry, use a hollow cylinder. For objects with planar symmetry, use a box or cylinder.

4. Write Gauss's Law:

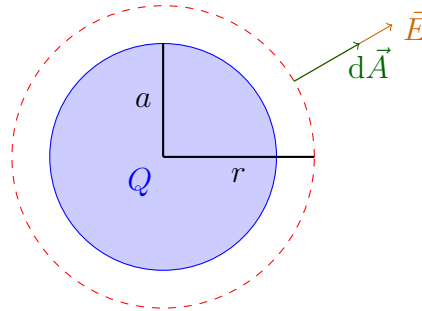
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

5. Use symmetry to solve the left hand side. Always pick a Gaussian surface so that  $\vec{E}$  is parallel to  $d\vec{A}$  so that  $\vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}|$ .
6. Also argue via symmetry that  $|\vec{E}|$  is independent of integrating variables so that  $\int |\vec{E}| |d\vec{A}| = |\vec{E}| \oint |d\vec{A}|$ .

7. Solve the integral for the area.  $|\vec{E}| \oint d\vec{A} = |\vec{E}|A$ . For a shell,  $A = 4\pi r^2$ . For the side of a cylindrical surface,  $A = 2\pi rh$ . For the top and bottom of a cylindrical surface,  $A = \pi r^2$ .
8. Calculate  $Q_{enc}$ , this can be easy or hard depending on the problem. Remember that  $Q = \rho V$ .
9. Solve for  $|\vec{E}|$ .

## Gauss's Law for a Spherical Solid

Case  $r > a$ :



Gaussian surface

$$\oint \vec{E} \cdot d\vec{A} = \int |\vec{E}| |d\vec{A}| = |\vec{E}| \int |d\vec{A}| = |\vec{E}|A = |\vec{E}|4\pi r^2$$

We can make this argument because  $\vec{E}$  and  $d\vec{A}$  are both radial and parallel to each other. The electric field is also constant over  $r$  by spherical symmetry, so we can bring it outside the integral. Since we are working with a Gaussian surface whose radius  $r$  is greater than the radius of the solid  $a$ , the enclosed charge is just the total charge.

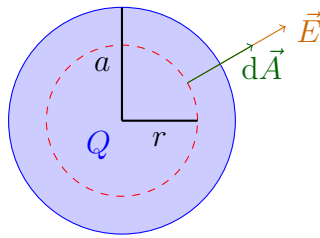
$$|\vec{E}|4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$|\vec{E}| = \frac{1}{4\pi r^2} \frac{Q}{\epsilon_0}$$

$$\vec{E} = \frac{1}{4\pi r^2} \frac{Q}{\epsilon_0} \hat{r}$$

Note that the formula for the electric field in this case is just like that of a point charge.

Case  $r < a$ :



$$\oint \vec{E} \cdot d\vec{A} = |\vec{E}|4\pi r^2$$

This argument can be made for the same reasons as above. The enclosed charge must be expressed in terms of  $r$  now. For this problem, we can assume the charge is uniform, which means it can be expressed using the following ratio:

$$\begin{aligned}\rho &= \frac{Q_{total}}{V_{total}} = \frac{Q_{enc}}{V_{enc}} \\ Q_{enc} &= \frac{Q_{total}V_{enc}}{V_{total}} \\ &= \frac{Q}{\frac{4}{3}\pi a^3} \frac{4}{3}\pi r^3 \\ &= \frac{Qr^3}{a^3}\end{aligned}$$

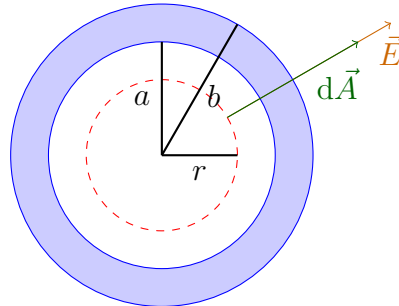
Now we can solve for  $|\vec{E}|$ :

$$\begin{aligned}|\vec{E}|4\pi r^2 &= \frac{Q_{enc}}{\epsilon_0} = \frac{Qr^3}{\epsilon_0 a^3} \\ |\vec{E}| &= \frac{1}{4\pi r^2} \frac{Qr^3}{\epsilon_0 a^3} = \frac{1}{4\pi} \frac{Qr}{\epsilon_0 a^3}\end{aligned}$$

From these two results, we can see that the electric field increases linearly inside the solid up to  $a$ , and then decreases proportionally to  $\frac{1}{r^2}$ .

## Gauss's Law for a Spherical Hollow Shell

Case  $r < a$ :



$$\oint \vec{E} \cdot d\vec{A} = |\vec{E}|4\pi r^2$$

The same argument as above can be applied here. Since the Gaussian surface is inside the solid, it encloses no charge, meaning  $Q_{enc} = 0$ .

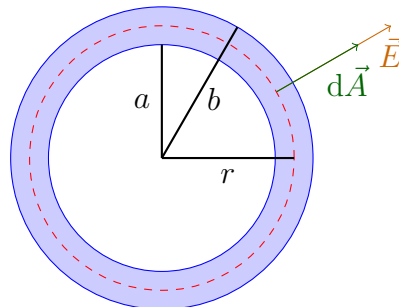
$$|\vec{E}|4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = 0$$

$$|\vec{E}| = 0$$

$$\vec{E} = \vec{0}$$

The electric field anywhere inside the solid is zero.

Case  $a < r < b$ :



$$\oint \vec{E} \cdot d\vec{A} = |\vec{E}|4\pi r^2$$

The same argument as above can be applied here. The enclosed charge must be expressed in terms of  $r$  now.

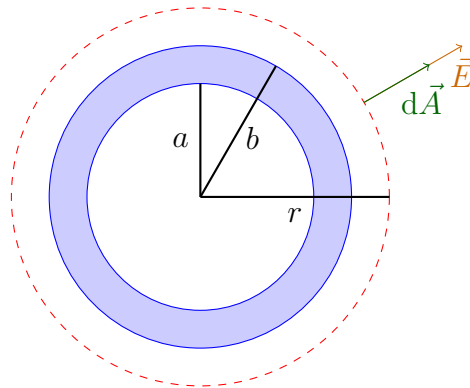
$$Q_{enc} = \int \rho \, dV = \rho V = \rho \left( \frac{4}{3} \pi r^3 - \frac{4}{3} \pi a^3 \right) = \frac{4}{3} \pi \rho (r^3 - a^3)$$

Now we can plug this back into Gauss's Law:

$$\begin{aligned} |\vec{E}| 4\pi r^2 &= \frac{Q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0} \frac{4}{3} \pi \rho (r^3 - a^3) \\ |\vec{E}| &= \frac{1}{4\pi r^2 \epsilon_0} \frac{4}{3} \pi \rho (r^3 - a^3) \\ &= \frac{\rho}{3\epsilon_0} \frac{r^3 - a^3}{r^2} \end{aligned}$$

Note that inside the solid, the charge increases linearly.

**Case  $r > b$ :**



$$\oint \vec{E} \cdot d\vec{A} = |\vec{E}| 4\pi r^2$$

This same argument as above can be applied here. Like the spherical solid, since the Gaussian surface is larger than the solid, the enclosed charge is just the total charge.

$$\begin{aligned} |\vec{E}| 4\pi r^2 &= \frac{Q_{enc}}{\epsilon_0} = \frac{Q}{\epsilon_0} \\ |\vec{E}| &= \frac{1}{4\pi r^2} \frac{Q}{\epsilon_0} \\ \vec{E} &= \frac{1}{4\pi r^2} \frac{Q}{\epsilon_0} \hat{r} \end{aligned}$$

Notice that this formula is again very much like that of a point charge. For this spherical hollow, it can also be written as:

$$|\vec{E}| = \frac{\rho}{3\epsilon_0} \frac{b^3 - a^3}{r^2}$$

## Important Relations

$$\begin{aligned}\lambda &= \frac{Q}{L} \\ \sigma &= \frac{Q}{A} \\ \rho &= \frac{Q}{V} = \frac{Q}{AL} = \frac{\lambda}{A} = \frac{\sigma}{L} \\ Q_{total} &= \int \rho \, dV\end{aligned}$$

## Rules for Conductors

The Gauss Law applications above apply for insulating solids. For conductors, we must observe a few rules:

- Conductors (metals) allow for the flow of charge.
- $Q_{enc} = 0$  inside the conductor since all charges are on the surface. Therefore the electric field is also zero inside a conductor.

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)