University Physics 1A

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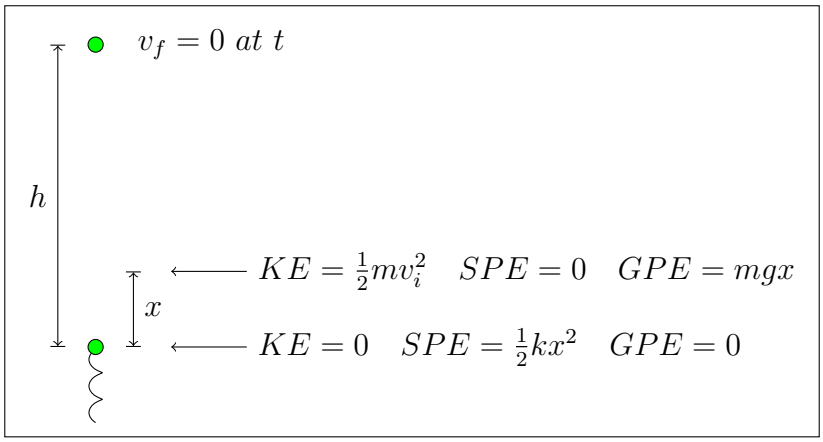
Conservation of Energy

A ball of mass m is launched from a vertical spring of spring constant k. It reaches its maximum height above the equilibrium position of the spring in a time t after launch. Say you are asked to find the initial compression of the spring relative to equilibrium in terms of the variables provided and g. Note that the maximum height is not a provided value.



$$\begin{aligned} 0 = v_f &= v_i + at \\ v_i &= -at \\ \frac{1}{2}kx^2 &= \frac{1}{2}mv_i^2 + mgx \\ 0 &= \frac{1}{2}kx^2 - mgx - \frac{1}{2}m(-at)^2 \end{aligned}$$

$$
\begin{aligned}
0 &= \left(\frac{1}{2}k\right)x^2 + (-mg)x - \frac{1}{2}ma^2t^2 \\
a &= \frac{1}{2}k \\
b &= -mg \\
c &= -\frac{1}{2}ma^2t^2 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-mg \pm \sqrt{(mg)^2 + kma^2t^2}}{k}
\end{aligned}
$$

In a new exciting ride, a roller-coaster car passed the top of one hill, then rolls to the bottom on a track. It then heads up a flat ramp and flies off the end of the ramp. Suppose that the car has a speed of 0.9m/s at the top of the hill, which is 8.0m off the ground. Say the ramp has a height of 1.0m above the ground and makes an angle of 30.0° with the horizontal. Model this problem so that air resistance and friction are negligible. Use ideas of energy and kinematics to find the following:

1. the speed of the car as it leaves the ramp.

$$
\begin{aligned}
KE_i + PE_i &= KE_f + PE_f \\
\frac{1}{2}mv_i^2 + mgh_i &= \frac{1}{2}mv_f^2 + mgh_f \\
\frac{1}{2}(9^2) + g(8) &= \frac{1}{2}v_f^2 + g(1) \\
v_f &= 14.77\frac{m}{s}
\end{aligned}
$$

2. the speed of the car when it reaches maximum height in the air after leaving the ramp.

$$
\begin{aligned}
v_y &= 0 \\
v_x &= v\cos\theta \\
&= 14.77\cos(30) \\
&= 12.79\frac{m}{s}
\end{aligned}
$$

3. the maximum height that the car reaches above the ground after leaving the

ramp.

$$\begin{aligned} v_f^2 &= v_i^2 + 2gd \\ 0 &= (14.8\sin(30))^2 + 2(-9.8)d \\ d &= 2.8m \\ d + x_0 &= 2.8 + 1 = 3.8m \end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech