

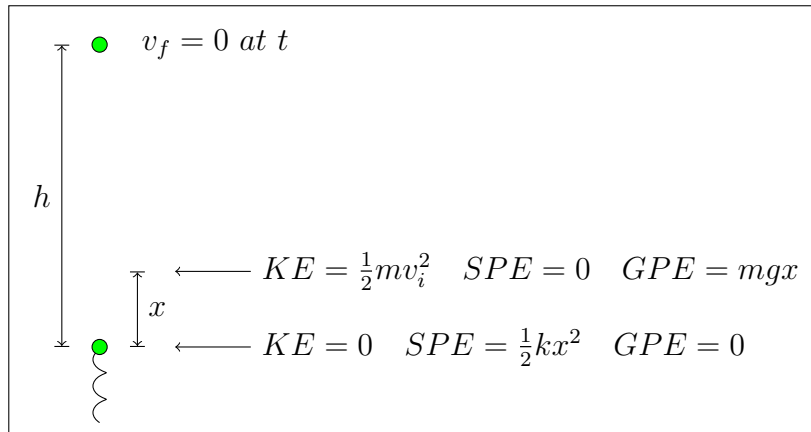
University Physics 1A

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Conservation of Energy

A ball of mass m is launched from a vertical spring of spring constant k . It reaches its maximum height above the equilibrium position of the spring in a time t after launch. Say you are asked to find the initial compression of the spring relative to equilibrium in terms of the variables provided and g . Note that the maximum height is not a provided value.



$$\begin{aligned}0 &= v_f = v_i + at \\v_i &= -at \\ \frac{1}{2}kx^2 &= \frac{1}{2}mv_i^2 + mgx \\ 0 &= \frac{1}{2}kx^2 - mgx - \frac{1}{2}m(-at)^2\end{aligned}$$

$$\begin{aligned}
0 &= \left(\frac{1}{2}k\right)x^2 + (-mg)x - \frac{1}{2}ma^2t^2 \\
a &= \frac{1}{2}k \\
b &= -mg \\
c &= -\frac{1}{2}ma^2t^2 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-mg \pm \sqrt{(mg)^2 + kma^2t^2}}{k}
\end{aligned}$$

In a new exciting ride, a roller-coaster car passed the top of one hill, then rolls to the bottom on a track. It then heads up a flat ramp and flies off the end of the ramp. Suppose that the car has a speed of 0.9m/s at the top of the hill, which is 8.0m off the ground. Say the ramp has a height of 1.0m above the ground and makes an angle of 30.0° with the horizontal. Model this problem so that air resistance and friction are negligible. Use ideas of energy and kinematics to find the following:

1. the speed of the car as it leaves the ramp.

$$\begin{aligned}
KE_i + PE_i &= KE_f + PE_f \\
\frac{1}{2}mv_i^2 + mgh_i &= \frac{1}{2}mv_f^2 + mgh_f \\
\frac{1}{2}(9^2) + g(8) &= \frac{1}{2}v_f^2 + g(1) \\
v_f &= 14.77 \frac{m}{s}
\end{aligned}$$

2. the speed of the car when it reaches maximum height in the air after leaving the ramp.

$$\begin{aligned}
v_y &= 0 \\
v_x &= v \cos \theta \\
&= 14.77 \cos(30) \\
&= 12.79 \frac{m}{s}
\end{aligned}$$

3. the maximum height that the car reaches above the ground after leaving the

ramp.

$$v_f^2 = v_i^2 + 2gd$$

$$0 = (14.8 \sin(30))^2 + 2(-9.8)d$$

$$d = 2.8m$$

$$d + x_0 = 2.8 + 1 = 3.8m$$

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech