Probability and Statistics

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Probability and Statistics: January 2017 - May 2017

Intervals Based on a Normal Population Distribution

Assumption in 7.3:

- population: normal
- population standard deviation σ: unknown
- n: no need to be large

Review 7.1:

- population: normal
- population standard deviation σ: known
- n: no need to be large
- confidence interval for $\mu = E(X_1) = E(X_2) \ldots$

Review 7.2:

- population: any
- population standard deviation σ: unknown
- n: large
- confidence interval for $\mu = E(X_1)$

If the number of repetitions goes to infinity, $100(1 - \alpha)\%$ of the intervals contain μ. Each interval is obtained from observed values of $X_1, X_2, \ldots, X_n : x_1, x_2, \ldots, x_n$. Each interval is calculated from the observed sample proportion $\frac{x}{n}$ where x is the observed number of successes in the sample of size n. Review 7.3:

- population: normal

- population standard deviation σ: unknown

- n: no need to be large

Theorem

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

where $\overline{X}$ is the random variable for the sample mean, μ is the population mean, S is the random variable for the sample standard deviation, and n is the sample size. This random variable T has t_{n-1} distribution, or a t distribution with $n - 1$ degrees of freedom.

Proposition: $100(1 - \alpha)\%$ confidence interval for μ is:

$$\overline{x} \pm t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}}$$

where $\overline{x}$ is the observed sample mean and s is the observed sample standard deviation. Upper and lower confidence bounds for μ are

$$\overline{x} + t_{\alpha, n-1} \cdot \frac{s}{\sqrt{n}}$$

$$\overline{x} - t_{\alpha, n-1} \cdot \frac{s}{\sqrt{n}}$$

Prediction Interval

- population: normal

- population standard deviation σ: unknown

- n: no need to be large

A prediction interval (PI) for a single observation to be selected from a normal population distribution is:

$$\overline{x} \pm t_{\alpha/2, n-1} \cdot s \sqrt{1 + \frac{1}{n}}$$

The confidence level is $100(1 - \alpha)\%$.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech