

Linear Algebra: Homework 1

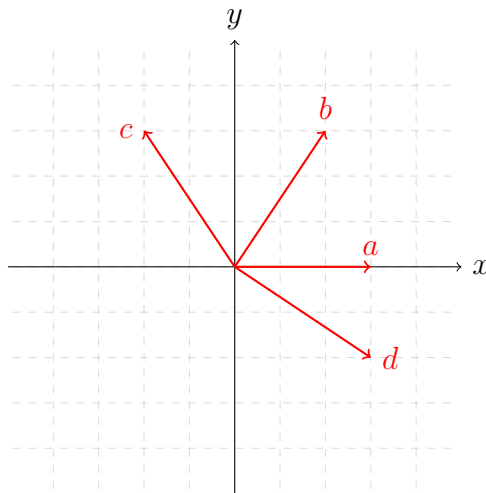
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Exercise 1

Draw the following vectors in standard position in \mathbb{R}^2 :

$$a = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad c = \begin{bmatrix} -2 \\ 3 \end{bmatrix} \quad d = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$



Exercise 3

Draw the following vectors in standard position in \mathbb{R}^3 :

$$a = [0, 2, 0]$$

$$b = [3, 2, 1]$$

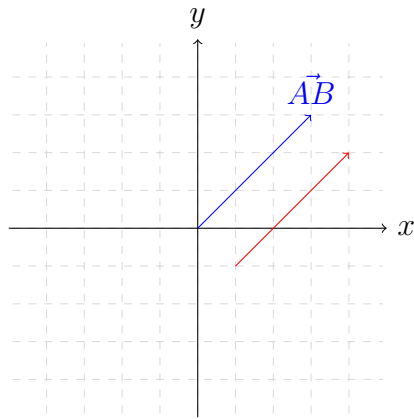
$$c = [1, -2, 1]$$

$$d = [-1, -1, -2]$$

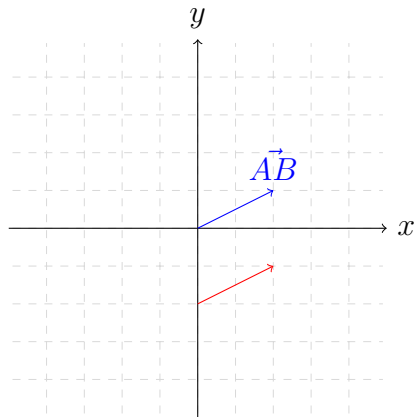
Exercise 5

For each of the following pairs of points, draw the vector \vec{AB} . Then compute and redraw \vec{AB} as a vector in standard position.

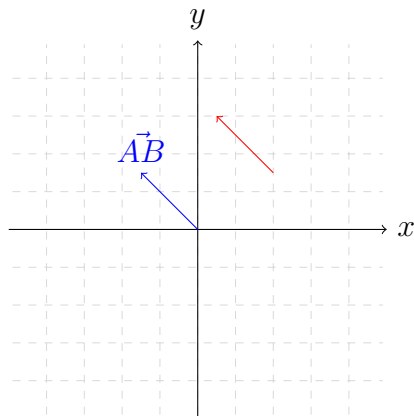
$$A = (1, -1), B = (4, 2)$$



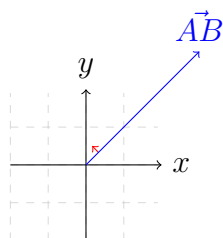
$$A = (0, -2), B = (2, -1)$$



$$A = (2, \frac{3}{2}), B = (\frac{1}{2}, 3)$$



$$A = (\frac{1}{3}, \frac{1}{3}), B = (\frac{1}{6}, \frac{1}{2})$$



Exercise 7

Compute the indicated vectors from Exercise 1 and show how the results can be obtained geometrically.

$$\begin{aligned}a + b &= \langle 3, 0 \rangle + \langle 2, 3 \rangle \\ &= \langle 5, 3 \rangle\end{aligned}$$

Exercise 9

Compute the indicated vectors from Exercise 1 and show how the results can be obtained geometrically.

$$\begin{aligned}d - c &= \langle 3, -2 \rangle - \langle -2, 3 \rangle \\ &= \langle 5, -5 \rangle\end{aligned}$$

Exercise 11

Compute the indicated vectors from Exercise 3.

$$\begin{aligned}2a + 3c &= 2\langle 0, 2, 0 \rangle + 2\langle 1, -2, 1 \rangle \\ &= \langle 0, 4, 0 \rangle + \langle 2, -4, 2 \rangle \\ &= \langle 2, 0, 2 \rangle\end{aligned}$$

Exercise 13

Find the components of the vectors $u, v, u + v, u - v$ where u and v are as shown in Figure 1.23.

$$\begin{aligned}u &= \langle \cos(60), \sin(60) \rangle = \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle \\ v &= \langle \cos(30), \sin(30) \rangle = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle \\ u + v &= \left\langle \frac{1 + \sqrt{3}}{2}, \frac{1 + \sqrt{3}}{2} \right\rangle \\ u - v &= \left\langle \frac{1 - \sqrt{3}}{2}, \frac{\sqrt{3} - 1}{2} \right\rangle\end{aligned}$$

Exercise 15

$$\begin{aligned}2(a - 3b) + 3(2b + a) &= 2a - 6b + 6b + 3a \\ &= 5a\end{aligned}$$

Exercise 17

$$\begin{aligned}x - a &= 2(x - 2a) \\ x - a &= 2x - 4a \\ x &= 2x - 4a + a \\ -x &= -3a \\ x &= 3a\end{aligned}$$

Exercise 24

Give algebraic proofs of properties (d) through (g) of Theorem 1.1:

Proof of (d):

$$\begin{aligned}\vec{u} + (-\vec{u}) &= \langle u_1, u_2, \dots, u_n \rangle + \langle -u_1, -u_2, \dots, -u_n \rangle \\ &= \langle u_1 - u_1, u_2 - u_2, \dots, u_n - u_n \rangle \\ &= \langle 0, 0, \dots, 0 \rangle = \vec{0}\end{aligned}$$

Proof of (e):

$$\begin{aligned}c(\vec{u} + \vec{v}) &= c(\langle u_1, u_2, \dots, u_n \rangle + \langle v_1, v_2, \dots, v_n \rangle) \\ &= c(\langle u_1 + v_1, u_2 + v_2, \dots, u_n + v_n \rangle) \\ &= \langle c(u_1 + v_1), c(u_2 + v_2), \dots, c(u_n + v_n) \rangle \\ &= \langle cu_1 + cv_1, cu_2 + cv_2, \dots, cu_n + cv_n \rangle \\ &= \langle cu_1, cu_2, \dots, cu_n \rangle + \langle cv_1, cv_2, \dots, cv_n \rangle \\ &= c\langle u_1, u_2, \dots, u_n \rangle + c\langle v_1, v_2, \dots, v_n \rangle \\ &= c\vec{u} + c\vec{v}\end{aligned}$$

Proof of (f):

$$\begin{aligned}(c + d)\vec{u} &= (c + d)\langle u_1, \dots, u_n \rangle \\ &= \langle (c + d)u_1, \dots, (c + d)u_n \rangle \\ &= \langle cu_1 + du_1, \dots, cu_n + du_n \rangle \\ &= \langle cu_1, \dots, cu_n \rangle + \langle du_1, \dots, du_n \rangle \\ &= c\vec{u} + d\vec{u}\end{aligned}$$

Proof of (g):

$$\begin{aligned}c(d\vec{u}) &= (c)\langle du_1, \dots, du_n \rangle \\ &= (c)(d)\langle u_1, \dots, u_n \rangle \\ &= (cd)\langle u_1, \dots, u_n \rangle \\ &= (cd)\vec{u}\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanagerd.tech