Multivariable and Vector Calculus: Homework 4

Alvin Lin

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Section 13.3

Exercise 3

Find the length of the curve.

$$\overrightarrow{r(t)} = \sqrt{2}t\hat{i} + \mathrm{e}^t\hat{j} + \mathrm{e}^{-t}\hat{k} \quad 0 \le t \le 1$$

$$\begin{aligned}
L &= \int_0^1 |\langle\sqrt{2}, \mathrm{e}^t, -\mathrm{e}^{-t}\rangle| \, \mathrm{d}t \\
&= \int_0^1 \sqrt{2 + \mathrm{e}^{2t} + \mathrm{e}^{-2t}} \, \mathrm{d}t \\
&= \int_0^1 \sqrt{(\mathrm{e}^t + \mathrm{e}^{-t})^2} \, \mathrm{d}t \\
&= \mathrm{e}^t - \mathrm{e}^{-t}\Big]_0^1 \\
&= \mathrm{e} - \frac{1}{\mathrm{e}} - (1 - 1) \\
&= \mathrm{e} - \frac{1}{\mathrm{e}}
\end{aligned}$$

Exercise 5

Find the length of the curve.

$$\overrightarrow{r(t)} = \hat{i} + t^2\hat{j} + t^3\hat{k} \quad 0 \le t \le 1$$

$$\begin{aligned}
L &= \int_0^1 |\langle 0, 2t, 3t^2 \rangle| \;\mathrm{d}t \\
&= \int_0^1 \sqrt{0 + 4t^2 + 9t^4} \;\mathrm{d}t \\
&= \int_0^1 \sqrt{t^2}\sqrt{9t^2 + 4} \;\mathrm{d}t \\
&= \int_0^1 t\sqrt{9t^2 + 4} \;\mathrm{d}t \\
Let : u &= 9t^2 + 4 \\
\mathrm{d}u &= 18t \;\mathrm{d}t \\
&= \int_4^{13} t\sqrt{u}\frac{\mathrm{d}u}{18t} \\
&= \frac{1}{18}\int_4^{13} \sqrt{u} \;\mathrm{d}u \\
&= \frac{1}{18}\frac{2}{3}u^{\frac{3}{2}}\Big]_4^{13} \\
&= \frac{1}{27}(13^{\frac{3}{2}} - 4^{\frac{3}{2}}) \\
&= \frac{1}{27}(13^{\frac{3}{2}} - 8)
\end{aligned}$$

Section 13.4

Exercise 11

Find the velocity, acceleration, and speed of a particle with the given position function.

$$\overrightarrow{r(t)} = \sqrt{2}t\hat{\imath} + \mathrm{e}^t\hat{\jmath} + \mathrm{e}^{-t}\hat{k}$$

$$\begin{aligned}
\overrightarrow{r'(t)} &= \sqrt{2}\hat{\imath} + \mathrm{e}^t\hat{\jmath} - \mathrm{e}^{-t}\hat{k} \\
\overrightarrow{r''(t)} &= \mathrm{e}^t\hat{\jmath} + \mathrm{e}^{-t}\hat{k}
\end{aligned}$$

Exercise 15

Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position.

$$\overrightarrow{a(t)} = 2\hat{\imath} + 2t\hat{k} \quad \overrightarrow{v(0)} = 3\hat{\imath} - \hat{\jmath} \quad \overrightarrow{r(0)} = \hat{\jmath} + \hat{k}$$

$$\begin{aligned}
\overrightarrow{v(t)} &= (2t + c_1)\hat{\imath} + c_2\hat{\jmath} + (t^2 + c_3)\hat{k} \\
\overrightarrow{v(0)} &= 3\hat{\imath} - \hat{\jmath} \\
\overrightarrow{v(t)} &= (2t + 3)\hat{\imath} - \hat{\jmath} + t^2\hat{k} \\
\overrightarrow{r(t)} &= (t^2 + 3t + c_1)\hat{\imath} + (-t + c_2)\hat{\jmath} + (\frac{t^3}{3} + c_3)\hat{k} \\
\overrightarrow{r(0)} &= \hat{\jmath} + \hat{k} \\
\overrightarrow{r(t)} &= (t^2 + 3t)\hat{\imath} + (-t + 1)\hat{\jmath} + (\frac{t^3}{3} + 1)\hat{k}
\end{aligned}$$

Exercise 23

A projectile is fired with an initial speed of 200m/s and angle of elevation 60°. Find the range of the projectile, the maximum height reached, and the speed at impact.

$$
\begin{aligned}
d_y &= y_\circ + v_{y\circ}t + \frac{1}{2}at^2 \\
0 &= 0 + 200\sin(60)t - \frac{1}{2}(9.8)t^2 \\
t &= 0 \quad t = 35.347s \\
d_x &= x_\circ + v_{x\circ} + \frac{1}{2}at^2 \\
&= 200\text{7̧}s(60)(35.347) \\
&= 3534.1m \\
v_{yf} &= v_{yi} + at \\
0 &= 200\sin(60) - (9.8)t \\
t &= 17.674s \\
d_y &= 0 + 200\sin(60)(17.674) - \frac{1}{2}(9.8)(17.674^2) \\
&= 1530.6m \\
v_f &= 200m/s
\end{aligned}
$$

Exercise 25

A ball is thrown at an angle of 45° to the ground. If the ball lands 90m away, what was the initial speed of the ball?

$$
\begin{aligned}
d_x &= x_\circ + v_{x\circ}t + \frac{1}{2}at^2 \\
90 &= 0 + v\cos(45)t \\
t &= \frac{90}{v\cos(45)} \\
d_y &= y_\circ + v_{y\circ}t + \frac{1}{2}at^2 \\
0 &= 0 + v\sin(45)t - 4.9t^2 \\
&= v\sin(45)\frac{90}{v\cos(45)} - 4.9(\frac{90}{v\cos(45)})^2 \\
\frac{(4.9)(90^2)}{v^2\cos^2(45)} &= \frac{90\sin(45)}{\cos(45)} \\
\frac{(4.9)(90)}{v^2\cos(45)} &= \sin(45) \\
v &= \sqrt{\frac{(4.9)(90)}{\cos(45)\sin(45)}} \\
&= 29.69m/s
\end{aligned}
$$

Exercise 27

A rifle is fired with angle of elevation 36°. What is the muzzle speed if the maximum height of the bullet is 1600ft.

$$\begin{aligned}
(v_{yf})^2 &= (v_{yi})^2 + 2ad \\
0 &= (v_{yi})^2 - 2(9.8)(1600) \\
v_{yi} &= 177.08m/s \\
v\sin(36) &= 177.08m.s \\
v &= \frac{177.08m/s}{\sin(36)} = 301.27m/s
\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech