

Multivariable and Vector Calculus: Homework 4

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Section 13.3

Exercise 3

Find the length of the curve.

$$\vec{r}(t) = \sqrt{2}t\hat{i} + e^t\hat{j} + e^{-t}\hat{k} \quad 0 \leq t \leq 1$$

$$\begin{aligned} L &= \int_0^1 |\langle \sqrt{2}, e^t, -e^{-t} \rangle| dt \\ &= \int_0^1 \sqrt{2 + e^{2t} + e^{-2t}} dt \\ &= \int_0^1 \sqrt{(e^t + e^{-t})^2} dt \\ &= \left[e^t - e^{-t} \right]_0^1 \\ &= e - \frac{1}{e} - (1 - 1) \\ &= e - \frac{1}{e} \end{aligned}$$

Exercise 5

Find the length of the curve.

$$\vec{r}(t) = \hat{i} + t^2\hat{j} + t^3\hat{k} \quad 0 \leq t \leq 1$$

$$\begin{aligned}
L &= \int_0^1 |\langle 0, 2t, 3t^2 \rangle| dt \\
&= \int_0^1 \sqrt{0 + 4t^2 + 9t^4} dt \\
&= \int_0^1 \sqrt{t^2} \sqrt{9t^2 + 4} dt \\
&= \int_0^1 t \sqrt{9t^2 + 4} dt
\end{aligned}$$

$$\text{Let } u = 9t^2 + 4$$

$$du = 18t dt$$

$$\begin{aligned}
&= \int_4^{13} t \sqrt{u} \frac{du}{18t} \\
&= \frac{1}{18} \int_4^{13} \sqrt{u} du \\
&= \frac{1}{18} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_4^{13} \\
&= \frac{1}{27} (13^{\frac{3}{2}} - 4^{\frac{3}{2}}) \\
&= \frac{1}{27} (13^{\frac{3}{2}} - 8)
\end{aligned}$$

Section 13.4

Exercise 11

Find the velocity, acceleration, and speed of a particle with the given position function.

$$\vec{r}(t) = \sqrt{2}t\hat{i} + e^t\hat{j} + e^{-t}\hat{k}$$

$$\vec{r}'(t) = \sqrt{2}\hat{i} + e^t\hat{j} - e^{-t}\hat{k}$$

$$\vec{r}''(t) = e^t\hat{j} + e^{-t}\hat{k}$$

Exercise 15

Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position.

$$\vec{a}(t) = 2\hat{i} + 2t\hat{k} \quad \vec{v}(0) = 3\hat{i} - \hat{j} \quad \vec{r}(0) = \hat{j} + \hat{k}$$

$$\vec{v}(t) = (2t + c_1)\hat{i} + c_2\hat{j} + (t^2 + c_3)\hat{k}$$

$$\vec{v}(0) = 3\hat{i} - \hat{j}$$

$$\vec{v}(t) = (2t + 3)\hat{i} - \hat{j} + t^2\hat{k}$$

$$\vec{r}(t) = (t^2 + 3t + c_1)\hat{i} + (-t + c_2)\hat{j} + \left(\frac{t^3}{3} + c_3\right)\hat{k}$$

$$\vec{r}(0) = \hat{j} + \hat{k}$$

$$\vec{r}(t) = (t^2 + 3t)\hat{i} + (-t + 1)\hat{j} + \left(\frac{t^3}{3} + 1\right)\hat{k}$$

Exercise 23

A projectile is fired with an initial speed of 200m/s and angle of elevation 60° . Find the range of the projectile, the maximum height reached, and the speed at impact.

$$d_y = y_o + v_{y_o}t + \frac{1}{2}at^2$$

$$0 = 0 + 200 \sin(60)t - \frac{1}{2}(9.8)t^2$$

$$t = 0 \quad t = 35.347s$$

$$d_x = x_o + v_{x_o} + \frac{1}{2}at^2$$

$$= 200 \cos(60)(35.347)$$

$$= 3534.1m$$

$$v_{yf} = v_{yi} + at$$

$$0 = 200 \sin(60) - (9.8)t$$

$$t = 17.674s$$

$$d_y = 0 + 200 \sin(60)(17.674) - \frac{1}{2}(9.8)(17.674^2)$$

$$= 1530.6m$$

$$v_f = 200m/s$$

Exercise 25

A ball is thrown at an angle of 45° to the ground. If the ball lands 90m away, what was the initial speed of the ball?

$$d_x = x_o + v_{x_o}t + \frac{1}{2}at^2$$

$$90 = 0 + v \cos(45)t$$

$$t = \frac{90}{v \cos(45)}$$

$$d_y = y_o + v_{y_o}t + \frac{1}{2}at^2$$

$$0 = 0 + v \sin(45)t - 4.9t^2$$

$$= v \sin(45) \frac{90}{v \cos(45)} - 4.9 \left(\frac{90}{v \cos(45)} \right)^2$$

$$\frac{(4.9)(90^2)}{v^2 \cos^2(45)} = \frac{90 \sin(45)}{\cos(45)}$$

$$\frac{(4.9)(90)}{v^2 \cos(45)} = \sin(45)$$

$$v = \sqrt{\frac{(4.9)(90)}{\cos(45) \sin(45)}}$$

$$= 29.69m/s$$

Exercise 27

A rifle is fired with angle of elevation 36° . What is the muzzle speed if the maximum height of the bullet is 1600ft.

$$\begin{aligned}(v_{yf})^2 &= (v_{yi})^2 + 2ad \\ 0 &= (v_{yi})^2 - 2(9.8)(1600) \\ v_{yi} &= 177.08m/s \\ v \sin(36) &= 177.08m/s \\ v &= \frac{177.08m/s}{\sin(36)} = 301.27m/s\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech