Multivariable and Vector Calculus: Homework 3

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Section 12.6

Exercise 11

Use traces to sketch and identify the surface.

$$x = y^2 + 4z^2$$

$$\begin{array}{llll} x = 0 & x = 1 & x = 5 & y = 0 \\ 0 = y^2 + 4z^2 & 1 = y^2 + 4z^2 & 5 = y^2 + 4z^2 & x = 4z^2 \end{array}$$

The surface is a elliptic paraboloid expanding along the positive x-axis.

Exercise 13

Use traces to sketch and identify the surface.

$$x^2 = 4y^2 + z^2$$

$$\begin{array}{llll} x = 0 & x = 1 & x = 2 & \begin{array}{c} y = 0 \\ x^2 = 4z^2 \\ x = \pm 2z \end{array} \\ 0 = 4y^2 + z^2 & 1 = 4y^2 + z^2 & 4 = 4y^2 + z^2 & \end{array}$$

This surface is a cone defined by ellipses along the x-axis.

Exercise 15

Use traces to sketch and identify the surface.

$$9y^2 + 4z^2 = x^2 + 36$$

$$\begin{array}{ccccc} x = 0 & x = 1 & x = 2 & y = 0 & \begin{array}{c} y = 2 \\ 36 + 4z^2 = x^2 + 36 \\ 4z^2 - x^2 = 0 \end{array} \\ 9y^2 + 4z^2 = 36 & 9y^2 + 4z^2 = 37 & 9y^2 + 4z^2 = 40 & 4z^2 - x^2 = 36 & \end{array}$$

This surface is a hyperboloid of one sheet.

Exercise 47

Find an equation for the surface consisting of all points that are equidistant from the point (-1,0,0) and the plane $x = 1$. Identify the surface.

$$\text{Plane} \equiv 1x + 0y + 0z - 1 = 0$$

$$\begin{aligned} d_1 &= d_2 \\ \sqrt{(x+1)^2 + y^2 + z^2} &= \frac{|1x - 1|}{\sqrt{1^2}} \\ (x+1)^2 + y^2 + z^2 &= (x-1)^2 \\ x^2 + 2x + 1 + y^2 + z^2 &= x^2 - 2x + 1 \\ y^2 + z^2 &= -4x \end{aligned}$$

The surface is a series of expanding circles along the negative x-axis. The cross section of the circles is a parabola. The surface is a elliptic paraboloid.

Section 13.1

Exercise 31

At what points does the curve $r(\vec{t}) = t\hat{i} + (2t - t^2)\hat{k}$ intersect the the paraboloid $z = x^2 + y^2$?

$$\begin{aligned} \overrightarrow{r(t)} &= \begin{cases} x &= t \\ y &= 2t - t^2 \\ z &= 0 \end{cases} \\ z &= x^2 + y^2 \\ 0 &= t^2 + (2t - t^2)^2 \\ 0 &= t^2 + (4t^2 - 4t^3 + t^4) \\ t^4 - 4t^3 + 5t^2 &= 0 \\ t^2(t-4)(t-1) &= 0 \\ t &= 4 \quad t = 1 \quad t = 0 \\ \overrightarrow{r(0)} &= \langle 0, 0, 0 \rangle \\ \overrightarrow{r(1)} &= \langle 1, 1, 0 \rangle \\ \overrightarrow{r(4)} &= \langle 4, -8, 0 \rangle \end{aligned}$$

Exercise 41

Show that the curve with parametric equations $x = t^2, y = 1 - 3t, z = 1 + 3t^2$ passes through the points (1,4,0) and (9,-8,28) but not through the point (4,7,-6).

$$\begin{array}{llllll} 1 = t^2 & t = \pm 1 & 9 = t^2 & t = \pm 3 & 4 = t^2 & t = \pm 2 \\ 4 = 1 - 3t & t = -1 & -8 = 1 - 3t & t = 3 & 7 = 1 - 3t & t = -2 \\ 0 = 1 + t^3 & t = -1 & 28 = 1 + t^3 & t = 3 & -6 = 1 + t^3 & t = \sqrt[3]{7} \end{array}$$

The equation does not solve to the same t for the point (4,7,-6).

Exercise 43

Find a vector function that represents the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.

$$\begin{aligned} \sqrt{x^2 + y^2} &= 1 + y \\ x^2 + y^2 &= (1 + y)^2 \\ x^2 + y^2 &= 1 + 2y + y^2 \\ x^2 - 2y &= 1 \\ x &= \cos(t) \\ -2y &= \sin^2(t) \\ z &= 1 - \frac{\sin^2(t)}{2} \\ \overrightarrow{r(t)} &= \langle \cos(t), \frac{-\sin^2(t)}{2}, 1 - \frac{\sin^2(t)}{2} \rangle \end{aligned}$$

Exercise 45

Find a vector function that represents the curve of intersection of the hyperboloid $z = x^2 - y^2$ and the cylinder $x^2 + y^2 = 1$.

$$\begin{aligned} x &= \cos(t) \\ y &= \sin(t) \\ z &= x^2 - y^2 \\ &= \cos^2(t) - \sin^2(t) \\ \overrightarrow{r(t)} &= \langle \cos(t), \sin(t), \cos^2(t) - \sin^2(t) \rangle \end{aligned}$$

Section 13.2

Exercise 23

Find the parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

$$x = t^2 + 1 \quad y = 4\sqrt{t} \quad z = e^{t^2 - t} \ at \ (2, 4, 1)$$

$$x' = 2t \quad y' = \frac{2}{\sqrt{t}} \quad z' = (2t - 1)e^{t^2 - t}$$

$$x'(2) = 4 \quad y'(4) = 1 \quad z'(1) = e^0 = 1$$

$$l = \begin{cases} x &= 2 + 4t \\ y &= 4 + t \\ z &= 1 + t \end{cases}$$

Exercise 27

Find a vector equation for the tangent line to the curve of intersection of the cylinders $x^2 + y^2 = 25$ and $y^2 + z^2 = 20$ at the point (3,4,2).

$$\begin{aligned}
\frac{x^2}{25} &= \cos^2(t) \\
x &= 5\cos(t) \\
\frac{y^2}{25} &= \sin^2(t) \\
y &= 5\sin(t) \\
z^2 &= 20 - y^2 = 20 - 25\sin^2(t) \\
z &= \sqrt{20 - 25\sin^2(t)} \\
\overrightarrow{r(t)} &= \left\langle 5\cos(t), 5\sin(t), \sqrt{20 - 25\sin^2(t)} \right\rangle \\
\overrightarrow{r(x)} &= \langle 3, 4, 2 \rangle \\
\cos(x) &= \frac{3}{5} \\
\sin(x) &= \frac{4}{5} \\
\overrightarrow{r'(t)} &= \left\langle -5\sin(t), 5\cos(t), \frac{-25\sin(t)\cos(t)}{\sqrt{20 - 25\sin^2(t)}} \right\rangle \\
\overrightarrow{r'(x)} &= \left\langle -4, 3, \frac{-12}{\sqrt{4}} \right\rangle \\
&= \langle -4, 3, -6 \rangle \\
\overrightarrow{l(t)} &= \langle 3 - 4t, 4 + 3t, 2 - 6t \rangle
\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech