Multivariable and Vector Calculus: Homework 1

Alvin Lin

August 2016 - December 2016

Page 796

Exercise 3

Which of the points A(-4,0,-1), B(-3,1,-5), and C(2,4,6) is closest to the yz-plane? Which point lies in the xz-plane?

$$A \to 4 \quad B- > 3 \quad C- > 2$$

Point C is closest to the yz-plane. The y-component of A is 0, so it lies in the xz-plane.

Exercise 9

Find the lengths of the sides of the triangle PQR. Is it a right triangle? Is it an isosceles triangle?

$$P(3,-2,-3) \quad Q(7,0,1) \quad R(1,2,1)$$

$$\begin{aligned}
\overline{PQ} &= \sqrt{16+4+16} = \sqrt{36} = 6 \\
\overline{QR} &= \sqrt{36+4+0} = \sqrt{40} \\
\overline{PR} &= \sqrt{4+16+16} = \sqrt{36} = 6
\end{aligned}$$

$\triangle$ PQR is an isosceles triangle.

Exercise 13

Find the intersection of the sphere with center (-3,2,5) and radius 4.

$$(x+3)^2 + (y-2)^2 + (z-5)^2 = 16$$

This sphere intersects the yz-plane at $x = 0$:

$$(0+3)^2 + (y-2)^2 + (z-5)^2 = 16$$

$$(y-2)^2 + (z-5)^2 = 7$$

The intersection is a circle centered at (0,2,5) with radius $\sqrt{7}$.

Exercise 17

Show that the equation represents a sphere, and find its center and radius:

$$\begin{gathered}x^2 + y^2 + z^2 - 2x - 4y + 8z = 15 \\ x^2 - 2x + y^2 - 4y + z^2 + 8z = 15 \\ x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 + 8z + 16 = 15 + 1 + 4 + 16 \\ (x-1)^2 + (y-2)^2 + (z+4)^2 = 36\end{gathered}$$

This sphere is centered at (1,2,-4) and has a radius of 6.

Exercise 33

Describe in words the region of $\mathbb{R}^3$ represented by the equations or inequality.

$$x^2 + y^2 + z^2 = 4$$

This describes a hollow sphere centered at the origin with a radius 2 (excluding the inside of the sphere).

Exercise 37

Describe in words the region of $\mathbb{R}^3$ represented by the equations or inequality.

$$x^2 + z^2 \leq 9$$

This describes a solid cylinder centered at the origin with radius 9 extending infinitely in the y-direction.

Exercise 41

Write inequalities to describe the region: The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$.

$$r < x^2 + y^2 + z^2 < R$$

Page 805

Exercise 7

In the figure, the tip of c and the tail of d are both the midpoint of QR. Express c and d in terms of a and b.

$$d = \frac{b-a}{2}$$

$$b = \frac{a+b}{2}$$

Exercise 8

If the vectors in the figure satisfy $|u| = |v| = 1$ and $u + v + w = 0$, what is $|w|$?

$$\begin{aligned}
|w| &= \sqrt{(w_x)^2 + (w_y)^2} \\
&= \sqrt{(w_x)^2 + 0} \\
&= w_x \\
&= u_x + v_x \\
&= 1\cos 45 + 1\cos 45 \\
&= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \\
&= \sqrt{2}
\end{aligned}$$

Exercise 27

What is the angle between the given vector and the positive direction of the x-axis?

$$\hat{\imath} + \sqrt{3}\hat{\jmath}$$

$$\begin{aligned}
\tan\theta &= \frac{y}{x} \\
\theta &= \tan^{-1}\frac{y}{x} \\
&= \tan^{-1}\frac{\sqrt{3}}{1} \\
&= 60^\circ
\end{aligned}$$

Exercise 33

Find the magnitude of the resultant force and the angle it makes with the positive x-axis.

$$\begin{aligned}
\vec{r} &= \langle 200\cos -300, 200\sin 60 + 0\rangle \\
&= \langle -200, \frac{200\sqrt{3}}{2}\rangle \\
\theta &= \tan^{-1}|\frac{y}{x}| \\
&= \frac{\frac{200\sqrt{3}}{2}}{200} \\
&= \frac{\sqrt{3}}{2} \\
&\approx 40.9^\circ
\end{aligned}$$

This is the angle with the negative x-axis. The angle with the positive x-axis is:

$$180 - \theta = 139.1$$

Exercise 35

A woman walks due west on the deck of a ship at 3 mi/h. The ship is moving north at a speed of 22 mi/h. Find the speed and direction of the woman relative to the surface of the water.

$$\vec{v} = \langle -3, 22 \rangle$$
$$|\vec{v}| = \sqrt{9 + 484} = \sqrt{493} \approx 22.2$$
$$\theta = \tan^{-1} \frac{3}{22} \approx 7.765$$

She is traveling at approximately 22.2 mi/h at 7.765° west of north or 82.235° north of west.

Exercise 41

Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point (2,4).

$$\begin{aligned} y' &= 2x \\ y'(2) &= 2(2) = 4 \\ \vec{u} &= \langle 1, 4 \rangle \\ \frac{\vec{u}}{|\vec{u}|} &= \frac{\langle 1, 4 \rangle}{\sqrt{1 + 16}} \\ &= \left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle \end{aligned}$$

Exercise 43

If A, B, and C are the verticies of a triangle, find:

$$\vec{AB} + \vec{BC} + \vec{CA}$$
$$\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$$

Page 812

5

Find $\vec{a} \cdot \vec{b}$:

$$\vec{a} = \langle 4, 1\frac{1}{4} \rangle \quad \vec{b} = \langle 6, -3, -8 \rangle$$
$$\vec{a} \cdot \vec{b} = 24 + (-3) + (-2) = 19$$

9

$$|a| = 7 \quad |b| = 4$$

the angle between a and b is 30°:

$$\begin{aligned} \cos\theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \\ \vec{a} \cdot \vec{b} &= |\vec{a}||\vec{b}| \cos\theta \\ &= (7)(4)\cos 30 \\ &\approx 24.24 \end{aligned}$$

23a

Determine whether the given vectors are orthogonal, parallel, or neither.

$$\vec{a} = \langle 9, 3 \rangle \quad \vec{b} = \langle -2, 6 \rangle$$

$$\vec{a} \cdot \vec{b} = -18 + 18 = 0 \quad \therefore \vec{a} \perp \vec{b}$$

29

Find the acute angle between the lines.

$$2x - y = 3 \quad 3x + y = 7$$

$$y = 2x - 3 \quad y = -3x + 7$$

$$\begin{aligned}
\vec{a} &= \langle 1, 2 \rangle \\
\vec{b} &= \langle 1, -3 \rangle \\
|\vec{a}| &= \sqrt{5} \\
|\vec{b}| &= \sqrt{17} \\
\cos\theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \\
\cos\theta &= \frac{1 + (-6)}{\sqrt{5}\sqrt{17}} \\
\theta &= \cos^{-1}(\frac{-5}{\sqrt{85}}) \\
\theta &\approx 122.84 \\
180 - \theta &= 57.16
\end{aligned}$$

31

Find the acute angles between the curves at their points of intersection.

$$y = x^2 \quad y = x^3$$

$$y' = 2x \quad y' = 3x$$

Point of intersection: (0,0). Both lines have a slope of 0 at (0,0).

$$\theta = 0$$

47

If $\vec{a} = \langle 3, 0, -1 \rangle$, find a vector $\vec{b}$ such that $comp_{\vec{a}}\vec{b} = 2$.

$$\begin{aligned}
comp_{\vec{a}}\vec{b} &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \\
2 &= \frac{\langle 3, 0, -1 \rangle \cdot \vec{b}}{\sqrt{10}} \\
2\sqrt{10} &= 3b_x - 1b_z \\
\vec{b} &= \langle 0, 0, -2\sqrt{10} \rangle
\end{aligned}$$

51

A sled is pulled along a level path through snow by a rope. A 30-lb force acting at an angle of 40° above the horizontal moves the sled 80ft. Find the work done by the force.

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= 30\langle \cos 40, \sin 40 \rangle \cdot \langle 80, 0 \rangle \\ &= (30 \cos 40)(80) \\ &\approx 1838.5 ft \cdot pounds \end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech