

Multivariable and Vector Calculus: Homework 1

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Exercise 3

Which of the points A(-4,0,-1), B(-3,1,-5), and C(2,4,6) is closest to the yz-plane? Which point lies in the xz-plane?

$$A \rightarrow 4 \quad B \rightarrow 3 \quad C \rightarrow 2$$

Point C is closest to the yz-plane. The y-component of A is 0, so it lies in the xz-plane.

Exercise 9

Find the lengths of the sides of the triangle PQR. Is it a right triangle? Is it an isosceles triangle?

$$P(3, -2, -3) \quad Q(7, 0, 1) \quad R(1, 2, 1)$$

$$\overline{PQ} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$$

$$\overline{QR} = \sqrt{36 + 4 + 0} = \sqrt{40}$$

$$\overline{PR} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

$\triangle PQR$ is an isosceles triangle.

Exercise 13

Find the intersection of the sphere with center (-3,2,5) and radius 4.

$$(x + 3)^2 + (y - 2)^2 + (z - 5)^2 = 16$$

This sphere intersects the yz-plane at $x = 0$:

$$(0 + 3)^2 + (y - 2)^2 + (z - 5)^2 = 16$$

$$(y - 2)^2 + (z - 5)^2 = 7$$

The intersection is a circle centered at (0,2,5) with radius $\sqrt{7}$.

Exercise 17

Show that the equation represents a sphere, and find its center and radius:

$$\begin{aligned}x^2 + y^2 + z^2 - 2x - 4y + 8z &= 15 \\x^2 - 2x + y^2 - 4y + z^2 + 8z &= 15 \\x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 + 8z + 16 &= 15 + 1 + 4 + 16 \\(x - 1)^2 + (y - 2)^2 + (z + 4)^2 &= 36\end{aligned}$$

This sphere is centered at (1,2,-4) and has a radius of 6.

Exercise 33

Describe in words the region of \mathbb{R}^3 represented by the equations or inequality.

$$x^2 + y^2 + z^2 = 4$$

This describes a hollow sphere centered at the origin with a radius 2 (excluding the inside of the sphere).

Exercise 37

Describe in words the region of \mathbb{R}^3 represented by the equations or inequality.

$$x^2 + z^2 \leq 9$$

This describes a solid cylinder centered at the origin with radius 9 extending infinitely in the y-direction.

Exercise 41

Write inequalities to describe the region: The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$.

$$r < x^2 + y^2 + z^2 < R$$

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Exercise 7

In the figure, the tip of c and the tail of d are both the midpoint of QR . Express c and d in terms of a and b .

$$\begin{aligned}d &= \frac{b - a}{2} \\b &= \frac{a + b}{2}\end{aligned}$$

Exercise 8

If the vectors in the figure satisfy $|u| = |v| = 1$ and $u + v + w = 0$, what is $|w|$?

$$\begin{aligned} |w| &= \sqrt{(w_x)^2 + (w_y)^2} \\ &= \sqrt{(w_x)^2 + 0} \\ &= w_x \\ &= u_x + v_x \\ &= 1 \cos 45 + 1 \cos 45 \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \\ &= \sqrt{2} \end{aligned}$$

Exercise 27

What is the angle between the given vector and the positive direction of the x-axis?

$$\hat{i} + \sqrt{3}\hat{j}$$

$$\begin{aligned} \tan \theta &= \frac{y}{x} \\ \theta &= \tan^{-1} \frac{y}{x} \\ &= \tan^{-1} \frac{\sqrt{3}}{1} \\ &= 60^\circ \end{aligned}$$

Exercise 33

Find the magnitude of the resultant force and the angle it makes with the positive x-axis.

$$\begin{aligned} \vec{r} &= \langle 200 \cos -300, 200 \sin 60 + 0 \rangle \\ &= \langle -200, \frac{200\sqrt{3}}{2} \rangle \\ \theta &= \tan^{-1} \left| \frac{y}{x} \right| \\ &= \frac{\frac{200\sqrt{3}}{2}}{200} \\ &= \frac{\sqrt{3}}{2} \\ &\approx 40.9^\circ \end{aligned}$$

This is the angle with the negative x-axis. The angle with the positive x-axis is:

$$180 - \theta = 139.1$$

Exercise 35

A woman walks due west on the deck of a ship at 3 mi/h. The ship is moving north at a speed of 22 mi/h. Find the speed and direction of the woman relative to the surface of the water.

$$\begin{aligned}\vec{v} &= \langle -3, 22 \rangle \\ |\vec{v}| &= \sqrt{9 + 484} = \sqrt{493} \approx 22.2 \\ \theta &= \tan^{-1} \frac{3}{22} \approx 7.765\end{aligned}$$

She is traveling at approximately 22.2 mi/h at 7.765° west of north or 82.235° north of west.

Exercise 41

Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.

$$\begin{aligned}y' &= 2x \\ y'(2) &= 2(2) = 4 \\ \vec{u} &= \langle 1, 4 \rangle \\ \frac{\vec{u}}{|\vec{u}|} &= \frac{\langle 1, 4 \rangle}{\sqrt{1 + 16}} \\ &= \left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle\end{aligned}$$

Exercise 43

If A, B, and C are the vertices of a triangle, find:

$$\begin{aligned}\vec{AB} + \vec{BC} + \vec{CA} \\ \vec{AB} + \vec{BC} + \vec{CA} &= \vec{0}\end{aligned}$$

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Find $\vec{a} \cdot \vec{b}$:

$$\begin{aligned}\vec{a} &= \left\langle 4, 1\frac{1}{4} \right\rangle \quad \vec{b} = \langle 6, -3, -8 \rangle \\ \vec{a} \cdot \vec{b} &= 24 + (-3) + (-2) = 19\end{aligned}$$

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$$|a| = 7 \quad |b| = 4$$

the angle between a and b is 30° :

$$\begin{aligned}\cos \theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \\ \vec{a} \cdot \vec{b} &= |\vec{a}||\vec{b}| \cos \theta \\ &= (7)(4) \cos 30 \\ &\approx 24.24\end{aligned}$$

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Determine whether the given vectors are orthogonal, parallel, or neither.

$$\vec{a} = \langle 9, 3 \rangle \quad \vec{b} = \langle -2, 6 \rangle$$

$$\vec{a} \cdot \vec{b} = -18 + 18 = 0 \quad \therefore \vec{a} \perp \vec{b}$$

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Find the acute angle between the lines.

$$2x - y = 3 \quad 3x + y = 7$$

$$y = 2x - 3 \quad y = -3x + 7$$

$$\vec{a} = \langle 1, 2 \rangle$$

$$\vec{b} = \langle 1, -3 \rangle$$

$$|\vec{a}| = \sqrt{5}$$

$$|\vec{b}| = \sqrt{17}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos \theta = \frac{1 + (-6)}{\sqrt{5}\sqrt{17}}$$

$$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{85}}\right)$$

$$\theta \approx 122.84$$

$$180 - \theta = 57.16$$

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Find the acute angles between the curves at their points of intersection.

$$y = x^2 \quad y = x^3$$

$$y' = 2x \quad y' = 3x$$

Point of intersection: (0,0). Both lines have a slope of 0 at (0,0).

$$\theta = 0$$

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If $\vec{a} = \langle 3, 0, -1 \rangle$, find a vector \vec{b} such that $\text{comp}_{\vec{a}}\vec{b} = 2$.

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

$$2 = \frac{\langle 3, 0, -1 \rangle \cdot \vec{b}}{\sqrt{10}}$$

$$2\sqrt{10} = 3b_x - 1b_z$$

$$\vec{b} = \langle 0, 0, -2\sqrt{10} \rangle$$

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A sled is pulled along a level path through snow by a rope. A 30-lb force acting at an angle of 40° above the horizontal moves the sled 80ft. Find the work done by the force.

$$\begin{aligned}W &= \vec{F} \cdot \vec{d} \\&= 30\langle \cos 40, \sin 40 \rangle \cdot \langle 80, 0 \rangle \\&= (30 \cos 40)(80) \\&\approx 1838.5 \text{ft} \cdot \text{pounds}\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech