Multivariable and Vector Calculus

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Line Integral

$$\int_C f\ \mathrm{d}s = \int_a^b f|v'(t)|\ \mathrm{d}t$$

$$\int_C \vec{F}\ \mathrm{d}s = \int_C (\vec{F} \cdot \vec{T})\ \mathrm{d}s = \int_a^b \vec{F} \cdot v'(t)\ \mathrm{d}t$$

Find the work done against gravity by a 160 pound man going 3 revolutions up a silo $r = 20, h = 90$, losing 9 pounds out of 25 pounds of paint as it leaks from the bucket he is carrying up the silo.

$$\begin{aligned}
r(\vec{t}) &= \langle 20\cos(t), 20\sin(t), \frac{15t}{\pi}\rangle \\
\vec{F} &= \langle 0, 0, -185 + \frac{9t}{6\pi}\rangle \\
W &= \int_C \vec{F}\ \mathrm{d}r \\
&= \int_0^{6\pi} \vec{F} \cdot r(\vec{t})\ \mathrm{d}t \\
&= \int_0^{6\pi} -185 + \frac{9t}{6\pi}\ \mathrm{d}t
\end{aligned}$$

Fundamental Theorem for Line Integrals

$$\int_{C_{AB}} \overrightarrow{\nabla f} \cdot\ \mathrm{d}r = f(B) - f(A)$$

Note:

- If $\vec{F} = \overrightarrow{\nabla f}$, then F is a conservative vector field, and f is a potential function.
- If $\int_{C^1_{AB}} \overrightarrow{\nabla f}\ \mathrm{d}r = \int_{C^2_{AB}} \overrightarrow{\nabla f}\ \mathrm{d}r$ for all paths $A, B \in D$ and for all curves C_{AB}, then F is conservative in D.
- $\int_{C_{AA}} \overrightarrow{\nabla f}\ \mathrm{d}r = 0$
- If $F = \langle P(x,y), Q(x,y)\rangle$ and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \frac{\partial^2 P}{\partial y\ \mathrm{d}x} = \frac{\partial^2 Q}{\partial x\ \mathrm{d}y}$ are continuous, then F is conservative in D.
- D is open if and only if for every $(x_0, y_0) \in D$ there exist $\epsilon > 0$ such that the distance of radius ϵ and $(x_0, y_0) \in D$.
- D is connected if and only if for every $A, B \in D$ there exist $C_{AB} \subset D$.
- D is simply connected if and only if it is connected and every $C_{AA} \subset D$ has interior $C_{AA} \subset D$.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech