Multivariable and Vector Calculus

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Quadric Surfaces

Quadric surfaces are surfaces that can be described by quadratic equations in $\mathbb{R}^3$.

$$ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + iy + hz = j$$

We will only consider the cases where the coefficients d, e, f are zero and there are no mixed variables.

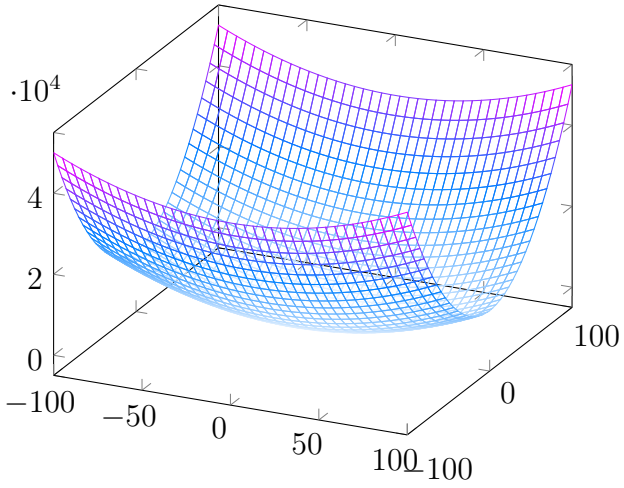
Example

Sketch $z = x^2 + 4y^2$. Trace:

$z = 0$	$z = 1$	$z = 5$	$z = -1$
$0 = x^2 + 4y^2$	$1 = x^2 + 4y^2$	$5 = x^2 + 4y^2$	$-1 = x^2 + 4y^2$

Starting from $z = 0$, this figure is a single point and expands as a series of ellipses along the positive z axis. If we take a slice along the yz-plane:

$$x = 0$$
$$z = 4y^2$$



Example

Sketch $x^2 + 4y^2 + 9z^2 = 9$:

$z = 0$	$z = \pm\frac{1}{3}$	$z = \pm 1$	$x = 0$
$x^2 + 4y^2 = 9$	$x^2 + 4y^2 = 8$	$x^2 + 4y^2 = 0$	$4y^2 + 9x^2 = 9$

This shape is ellipsoidal along the xy-plane.

Example

Sketch $x^2 - 4y^2 + 9z^2 = 9$:

$y = 0$	$y = \pm 1$	$y = \pm 5$	$z = 0$
$9 = x^2 + 9z^2$	$10 = x^2 + 9z^2$	$109 = x^2 + 9z^2$	$x^2 - 4y^2 = 9$

This shape looks like a series of expanding ellipses starting from $y = 0$. It is a hyperboloid.

Example

Sketch $x^2 - 4y^2 - 9z^2 = 9$:

$x = 0$	$x = \pm 3$	$x = \pm 5$	$x = \pm 8$	$x = 0$
$\emptyset$	$0 = 4y^2 + 9z^2$	$15 = 4y^2 + 9z^2$	$40 = 4y^2 + 9z^2$	$x^2 - 4y^2 = 9$

This shape is a hyperboloid of two sheets.

Example

Sketch $z^2 = x^2 + 4y^2$:

$$\begin{array}{ll} z = 0 & x = \pm 1 \\ 0 = x^2 + 4y^2 & 1 = x^2 + 4y^2 \end{array}$$

This is a series of expanding ellipses starting from $z = 0$. If we take $x = 0$, we get $z^2 = 4y^2$, which represents two intersecting lines or the cross section of the ellipses. We know from this that the radii of the ellipses increase linearly.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech