

Homework #4

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Discrete Math for Computing: January 2017 - May 2017

1

Determine whether each of the following expressions is true or false.

- (a) $x \in \{x\}$ True
- (b) $\{x\} \in \{\{x\}\}$ True
- (c) $\{x\} \subset \{x\}$ True
- (d) $\emptyset \subset \{x\}$ True
- (e) $\emptyset \in \{x\}$ False

2

Find the power set of each set below. Assume that a and b are distinct elements.

- (a) $P(\{a\}) = \{\emptyset, \{a\}\}$
- (b) $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
- (c) $P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

3

Let $A = \{a, b, c\}$, $B = \{c, d\}$ and $C = \{x, z\}$. Find

- (a) $A \times C = \{(a, c), (a, d), (b, c), (b, d), (c, c), (c, d)\}$
- (b) $C \times A \times B = \{(x, a, c), (x, a, d), (x, b, c), (x, b, d), (x, c, c), (x, c, d), (z, a, c), (z, a, d), (z, b, c), (z, b, d), (z, c, c), (z, c, d)\}$

$$(c) A \times B \times C = \{(a, c, x), (a, c, z), (a, d, x), (a, d, z), \\ (b, c, x), (b, c, z), (b, d, x), (b, d, z), (c, c, x), (c, c, z), (c, d, x), (c, d, z)\}$$

4

Suppose that

$$A = \{a, b, c, d, e, f\} \text{ and } B = \{a, b, c, d, e, f, g, h, i, j, k\}$$

Find the following

$$(a) A \cup B = \{a, b, c, d, e, f, g, h, i, j, k\}$$

$$(b) A - B = \emptyset$$

$$(c) A \cap B = \{a, b, c, d, e, f\}$$

$$(d) B - A = \{g, h, i, j, k\}$$

5

Suppose that A , B , and C are arbitrary sets. Show the following and justify:

$$(a) (A \cup B) \subseteq (A \cup B \cup C)$$

$$(A \cup B) = \{x \mid x \in A \vee x \in B\}$$

$$(A \cup B \cup C) = \{x \mid x \in A \vee x \in B \vee x \in C\}$$

Identity Law

$$(b) A - B = A \cap \overline{B}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \cap \overline{B} = \{x \mid x \in A \wedge x \notin B\}$$

$$(c) (A - B) - C = (A - C) - (B - C)$$

$$(A - B) - C = \{x \mid x \in A \wedge x \notin B \wedge x \notin C\}$$

$$(A - C) = \{x \mid x \in A \wedge x \notin C\}$$

$$(B - C) = \{x \mid x \in B \wedge x \notin C\}$$

$$(A - C) - (B - C) = \{x \mid x \in A \wedge x \notin C\} - \{x \mid x \in B \wedge x \notin C\}$$

$$(A - C) - (B - C) = \{x \mid x \in A \wedge x \notin B \wedge x \notin C\}$$

$$(d) \overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$$

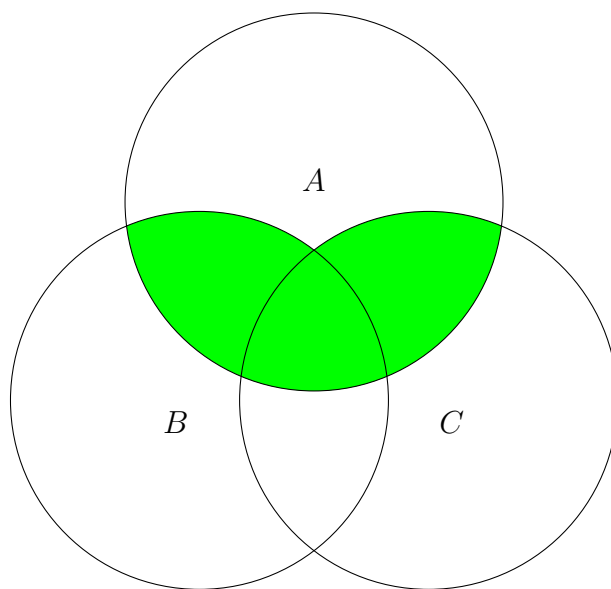
$$\begin{aligned} \overline{A \cup (B \cap C)} &= \overline{A} \cap \overline{B \cap C} \\ &= \overline{A} \cap (\overline{B} \cup \overline{C}) \\ &= (\overline{B} \cup \overline{C}) \cap \overline{A} \\ &= (\overline{C} \cup \overline{B}) \cap \overline{A} \end{aligned}$$

6

Suppose that A , B , and C are sets. Draw the Venn diagrams for each of the combinations below.

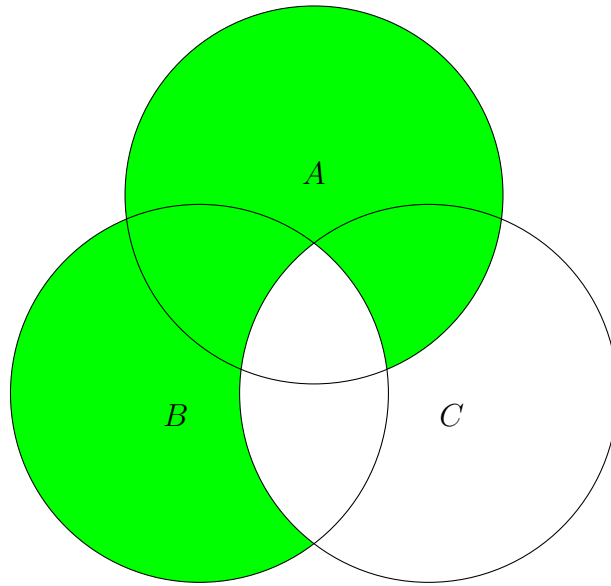
(a)

$$A \cap (B \cup C)$$



(b)

$$(A - B) \cup (A - C) \cup (B - C)$$



If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech