Sets

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Set Operations

Union

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

Let $A = \{1, 2\} \quad B = \{3, 4\}$:

$$A \cup B = \{1, 2, 3, 4\}$$

Intersection

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

Let $A = \{1, 2, 3\} \quad B = \{3, 4, 5\}$:

$$A \cap B = \{3\}$$

Set Difference

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

Let $A = \{a, b, c\} \quad B = \{b, c, d\}$:

$$A - B = \{a\}$$

Set Complements

Let u be the universal set. The **complement** of the set A, denoted $\overline{A}$, is the set $u - A$. An element belongs in $\overline{A}$ if $x \notin A$.

Set Identities

1. Identity Laws
    - $A \cap u = A$
    - $A \cup \varnothing = A$
2. Domination
    - $A \cup u = u$
    - $A \cap \varnothing = \varnothing$
3. Idempotent
    - $A \cup A = A$
    - $A \cap A = A$
4. Complementation
    - $\overline{(\overline{A})} = A$
5. Commutative
    - $A \cup B = B \cup A$
    - $A \cap B = B \cap A$
6. Associative
    - $A \cup (B \cup C) = (A \cup B) \cup C$
    - $A \cap (B \cap C) = (A \cap B) \cap C$
7. Distribution
    - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
    - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
8. De Morgan's
    - $\overline{A \cap B} = \overline{A} \cup \overline{B}$
    - $\overline{A \cup B} = \overline{A} \cap \overline{B}$
9. Absorption

- $A \cup (A \cap B) = A$
- $A \cap (A \cup B) = A$

10. Complement

    - $A \cup \overline{A} = u$
    - $A \cap \overline{A} = \varnothing$

Example

Show using set identities that:

$$\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$$

$$\begin{aligned}
\overline{A \cup (B \cap C)} &= \overline{A} \cap \overline{B \cap C} \quad (De\ Morgan's) \\
&= \overline{A} \cap (\overline{B} \cup \overline{C}) \quad (De\ Morgan's) \\
&= (\overline{B} \cup \overline{C}) \cap \overline{A} \quad (Commutative) \\
&= (\overline{C} \cup \overline{B}) \cap \overline{A} \quad (Commutative)
\end{aligned}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech