

# Nested Quantifiers

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## Nested Quantifiers

In section 1.4, we avoided nested quantifiers where one was in the scope of the other.

### Example

Assume that the domain of  $x$  and  $y$  are real numbers:

1.

$$\forall x \forall y (x + y = y + x)$$

For all real numbers  $x$  and  $y$ ,  $x + y = y + x$

2.

$$\forall x \exists y (x + y = 0)$$

For all real numbers  $x$ , there exists a  $y$  such that  $x + y = 0$ .

Translate the following into English:

$$\forall x \forall y ((x > 0) \wedge (y < 0)) \rightarrow (xy < 0)$$

For all real  $x$  and all real  $y$  if  $x > 0$  and  $y < 0$ , then  $xy < 0$ . The product of a positive real number and negative real number is negative.

## Order of Quantifiers

The order of quantifiers is essential, unless the quantifiers have all the same “type” (Either the quantifiers are all universal or all existential).

## Example

$$P(x, y) : x + y = y + x$$

What are the truth values when  $domain = \mathbb{R}$ ?

1.  $\forall x \forall y P(x, y)$  **True**
2.  $\forall y \forall x P(x, y)$  **True**

$$Q(x, y) : x + y = 0$$

Domain is  $\mathbb{R}$ , what are the truth values?

1.  $\exists y \forall x Q(x, y)$  There is a real number  $y$  such that for all real  $x$ ,  $x + y = 0$ . **False**
2.  $\forall x \exists y Q(x, y)$  For all real  $x$ , there is a real number  $y$  such that  $x + y = 0$ . **True**

## Example

Translate the following into a logical expression:

1. The sum of two positive integers is always positive.

$$\forall x \forall y ((x > 0) \wedge (y > 0)) \rightarrow (x + y > 0)$$

2. Every real number except 0 has a multiplicative inverse.

$$\forall x ((x \neq 0) \rightarrow \exists y (xy = 1))$$

## Example

Write  $\neg(\forall x \exists y (xy = 1))$  without any negations in the expression.

$$\exists x \neg(\exists y (xy = 1))$$

$$\exists x \forall y \neg(xy = 1)$$

$$\exists x \forall y (xy \neq 1)$$

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)