

Section 7.2

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Exercise 1

$$\begin{aligned} & \int \sin^2(x) \cos^3(x) \, dx \\ & \int \sin^2(x) \cos^2(x) \cos(x) \, dx \\ & \int \sin^2(x)(1 - \sin^2(x)) \cos(x) \, dx \end{aligned}$$

$$\text{Let : } u = \sin(x)$$

$$du = \cos(x) \, dx$$

$$\int u^2(1 - u^2) \, du$$

$$\int u^2 - u^4 \, du$$

$$\frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

$$\frac{1}{3} \sin^3(x) - \frac{1}{5} \sin^5(x) + C$$

Exercise 7

$$\begin{aligned} & \int_0^{\pi/2} \cos^2(\theta) \, d\theta \\ & \int_0^{\pi/2} \frac{\cos(2\theta) + 1}{2} \, d\theta \\ & \frac{1}{2} \int_0^{\pi/2} \cos(2\theta) + 1 \, d\theta \\ & \frac{1}{2} \left[\frac{1}{2} \sin(2\theta) + \theta \right]_0^{\pi/2} \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \left[0 + \frac{\pi}{2} - (0 + 0) \right] \\ = \frac{\pi}{4} \end{aligned}$$

Exercise 27

$$\begin{aligned} \int \tan^3(x) \sec(x) \, dx \\ \int \tan^2(x) \tan(x) \sec(x) \, dx \end{aligned}$$

Knowing this, we want to isolate a $\tan^2(x)$ because $\tan^2(x) = \sec^2(x) - 1$

$$\int (\sec^2(x) - 1) \tan(x) \sec(x) \, dx$$

$$\text{Let : } u = \sec(x)$$

$$du = \sec(x) \tan(x) \, dx$$

$$\int u^2 - 1 \, du$$

$$\frac{1}{3}u^3 - u + C$$

$$\frac{1}{3} \sec^3(x) - \sec(x) + C$$

Exercise 37

$$\begin{aligned} \int_{\pi/4}^{\pi/2} \cot^5(\phi) \csc^3(\phi) \, d\phi \\ \int_{\pi/4}^{\pi/2} \cot^4(\phi) \csc^2(\phi) \cot(\phi) \csc(\phi) \, d\phi \\ \int_{\pi/4}^{\pi/2} \cot^4(\phi) \csc^2(\phi) \cot(\phi) \csc(\phi) \, d\phi \\ \int_{\pi/4}^{\pi/2} (\csc^2(\phi) - 1)^2 \csc^2(\phi) \cot(\phi) \csc(\phi) \, d\phi \end{aligned}$$

For now we can solve it as an indefinite integral using u -substitution.

$$\text{Let : } u = \csc(\phi)$$

$$du = -\csc(\phi) \cot(\phi) \, d\phi$$

$$- \int (u^2 - 1)^2 u^2 \, du$$

$$\begin{aligned}
& - \int (u^2 - 1)^2 u^2 \, du \\
& - \int (u^4 - 2u^2 + 1)u^2 \, du \\
& - \int u^6 - 2u^4 + u^2 \, du \\
& - \left[\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3 \right] \\
& \left[-\frac{1}{5} \csc^7(\phi) + \frac{2}{5} \csc^5(\phi) - \frac{1}{3} \csc^3(\phi) \right]_{\pi/4}^{\pi/2}
\end{aligned}$$

Exercise 41

$$\begin{aligned}
& \int \sin(8x) \cos(5x) \, dx \\
& \int \frac{1}{2} \left[\sin(8x - 5x) + \sin(8x + 5x) \right] \, dx \\
& \frac{1}{2} \left[\int \sin(3x) \, dx + \int \sin(13x) \, dx \right] \\
& \frac{1}{6} \int 3 \sin(3x) \, dx + \frac{1}{26} \int 13 \sin(13x) \, dx \\
& -\frac{\cos(3x)}{6} - \frac{\cos(13x)}{26} + C
\end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech