Section 7.2

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Calculus II: August 2016 - December 2016

Exercise 1

$$\int \sin^2(x)\cos^3(x)\mathrm{d}x$$

$$\int \sin^2(x)\cos^2(x)\cos(x)\mathrm{d}x$$

$$\int \sin^2(x)(1-\sin^2(x))\cos(x)\mathrm{d}x$$

$$Let : u = \sin(x)$$

$$\mathrm{d}u = \cos(x)\mathrm{d}x$$

$$\int u^2(1-u^2)\mathrm{d}u$$

$$\int u^2 - u^4\mathrm{d}u$$

$$\frac{1}{3}u^3 - \frac{1}{5}u^5 + C$$

$$\frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + C$$

Exercise 7

$$\int_0^{\pi/2} \cos^2(\theta)\mathrm{d}\theta$$

$$\int_0^{\pi/2} \frac{\cos(2\theta)+1}{2}\mathrm{d}\theta$$

$$\frac{1}{2}\int_0^{\pi/2} \cos(2\theta)+1\mathrm{d}\theta$$

$$\frac{1}{2}\left[\frac{1}{2}\sin(2\theta)+\theta\right]_0^{\pi/2}$$

$$\frac{1}{2}\left[0+\frac{\pi}{2}-(0+0)\right]$$
$$=\frac{\pi}{4}$$

Exercise 27

$$\int \tan^3(x)\sec(x)\mathrm{d}x$$
$$\int \tan^2(x)\tan(x)\sec(x)\mathrm{d}x$$

Knowing this, we want to isolate a $\tan^2(x)$ because $\tan^2(x)=\sec^2(x)-1$

$$\int (\sec^2(x)-1)\tan(x)\sec(x)\mathrm{d}x$$
$$Let: u=\sec(x)$$
$$\mathrm{d}u=\sec(x)\tan(x)\mathrm{d}x$$
$$\int u^2-1\mathrm{d}u$$
$$\frac{1}{3}u^3-u+C$$
$$\frac{1}{3}\sec^3(x)-\sec(x)+C$$

Exercise 37

$$\int_{\pi/4}^{\pi/2}\cot^5(\phi)\csc^3(\phi)\mathrm{d}\phi$$
$$\int_{\pi/4}^{\pi/2}\cot^4(\phi)\csc^2(\phi)\cot(\phi)\csc(\phi)\mathrm{d}\phi$$
$$\int_{\pi/4}^{\pi/2}\cot^4(\phi)\csc^2(\phi)\cot(\phi)\csc(\phi)\mathrm{d}\phi$$
$$\int_{\pi/4}^{\pi/2}(\csc^2(\phi)-1)^2\csc^2(\phi)\cot(\phi)\csc(\phi)\mathrm{d}\phi$$

For now we can solve it as an indefinite integral using u-substitution.

$$Let: u=\csc(\phi)$$
$$\mathrm{d}u=-\csc(\phi)\cot(\phi)\mathrm{d}\phi$$
$$-\int (u^2-1)^2u^2\mathrm{d}u$$

$$-\int (u^2-1)^2 u^2 \mathrm{d}u$$

$$-\int (u^4-2u^2+1)u^2 \mathrm{d}u$$

$$-\int u^6-2u^4+u^2 \mathrm{d}u$$

$$-\left[\frac{1}{7}u^7-\frac{2}{5}u^5+\frac{1}{3}u^3\right]$$

$$\left[-\frac{1}{5}\csc^7(\phi)+\frac{2}{5}\csc^5(\phi)-\frac{1}{3}\csc^3(\phi)\right]_{\pi/4}^{\pi/2}$$

Exercise 41

$$\int \sin(8x)\cos(5x)\mathrm{d}x$$

$$\int \frac{1}{2}\left[\sin(8x-5x)+\sin(8x+5x)\right]\mathrm{d}x$$

$$\frac{1}{2}\left[\int \sin(3x)\mathrm{d}x+\int \sin(13x)\mathrm{d}x\right]$$

$$\frac{1}{6}\int 3\sin(3x)\mathrm{d}x+\frac{1}{26}\int 13\sin(13x)\mathrm{d}x$$

$$-\frac{\cos(3x)}{6}-\frac{\cos(13x)}{26}+C$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech