

Section 7.1

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Section 5.5

Exercise 3

$$\int x \cos(5x) dx$$

$$\text{Let : } u = x \quad dv = \cos(5x) dx$$

$$du = dx \quad v = \frac{\sin(5x)}{5}$$

$$\begin{aligned} \int x \cos(5x) dx &= \frac{x \sin(5x)}{5} - \frac{1}{5} \int \sin(5x) dx \\ &= \frac{x \sin(5x)}{5} - \frac{1}{5} \left(-\frac{\cos(5x)}{5} \right) + C \\ &= \frac{1}{5} x \sin(5x) + \frac{1}{25} \cos(5x) + C \end{aligned}$$

Exercise 11

$$\int t^4 \ln(t) dt$$

$$\text{Let : } u = \ln(t) \quad d(v) = t^4 dt$$

$$du = \frac{1}{t} dt \quad v = \frac{t^5}{5}$$

$$\begin{aligned} \int t^4 \ln(t) dt &= \frac{t^5 \ln t}{5} - \int \frac{t^5}{5t} dt \\ &= \frac{1}{5} t^4 \ln(t) - \frac{1}{5} \int t^4 dt = \frac{1}{5} t^4 \ln(t) - \frac{1}{5} \left[\frac{t^5}{5} \right] \\ &= \frac{1}{5} t^4 \left(\ln(t) - \frac{t}{5} \right) \end{aligned}$$

Exercise 17

$$\int e^{2\theta} \sin(3\theta) d(\theta)$$

$$\text{Let : } u = \sin(3\theta) \quad dv = e^{2\theta} d\theta$$

$$du = 3 \cos(3\theta) d\theta \quad v = \frac{e^{2\theta}}{2}$$

$$\int e^{2\theta} \sin(3\theta) d(\theta) = \frac{e^{2\theta} \sin(3\theta)}{2} - \int \frac{3e^{2\theta} \cos(3\theta)}{2} d\theta$$

$$\int e^{2\theta} \sin(3\theta) d(\theta) = \frac{e^{2\theta} \sin(3\theta)}{2} - \frac{3}{2} \int e^{2\theta} \cos(3\theta)$$

We must use integration by parts again to separate the integral.

$$\int e^{2\theta} \cos(3\theta)$$

$$\text{Let : } u = \cos(3\theta) \quad dv = e^{2\theta} d\theta$$

$$du = -3 \sin(3\theta) d\theta \quad v = \frac{e^{2\theta}}{2}$$

$$\int e^{2\theta} \cos(3\theta) = \frac{e^{2\theta} \cos(3\theta)}{2} - \int -\frac{3}{2} e^{2\theta} \sin(3\theta) d\theta$$

$$= \frac{e^{2\theta} \cos(3\theta)}{2} + \frac{3}{2} \int e^{2\theta} \sin(3\theta) d\theta$$

We can form the following equation from this:

$$\int e^{2\theta} \sin(3\theta) d\theta = \frac{e^{2\theta} \sin(3\theta)}{2} - \frac{3}{2} \left(\frac{e^{2\theta} \cos(3\theta)}{2} + \frac{3}{2} \int e^{2\theta} \sin(3\theta) d\theta \right)$$

$$\int e^{2\theta} \sin(3\theta) d\theta = \frac{e^{2\theta} \sin(3\theta)}{2} - \frac{3e^{2\theta} \cos(3\theta)}{4} - \frac{9}{4} \int e^{2\theta} \sin(3\theta) d\theta$$

$$\frac{13}{4} \int e^{2\theta} \sin(3\theta) d\theta = \frac{e^{2\theta} \sin(3\theta)}{2} - \frac{3e^{2\theta} \cos(3\theta)}{4}$$

$$= \int e^{2\theta} \sin(3\theta) d\theta = \frac{2e^{2\theta} \sin(3\theta)}{13} - \frac{3e^{2\theta} \cos(3\theta)}{13}$$

Exercise 19

$$\int z^3 e^z dz$$

$$\text{Let : } u = z^3 \quad dv = e^z dz$$

$$du = 3z^2 dz \quad v = e^z$$

$$\int z^3 e^z dz = z^3 e^z - \int 3z^2 e^z = z^3 e^z - 3 \int z^2 e^z$$

We can use integration by parts again to reduce the new integral.

$$\int z^2 e^z$$

$$\text{Let : } u = z^2 \quad dv = e^z dz$$

$$du = 2z dz \quad v = e^z$$

$$\int z^2 e^z = z^2 e^z - \int 2z e^z dz = z^2 e^z - 2 \int z e^z dz$$

We can use integration by parts one more time to remove the z from the integral.

$$\int z e^z dz$$

$$\text{Let : } u = z \quad dv = e^z dz$$

$$du = dz \quad v = e^z$$

$$\begin{aligned} \int z e^z dz &= z e^z - \int e^z dz \\ &= z e^z - e^z \end{aligned}$$

Now we put all this shit back together.

$$\begin{aligned} \int z^3 e^z dz &= z^3 e^z - 3 \int z^2 e^z \\ &= z^3 e^z - 3 \left(z^2 e^z - 2 \int z e^z dz \right) \\ &= z^3 e^z - 3z^2 e^z + 6 \int z e^z dz \\ &= z^3 e^z - 3z^2 e^z + 6(z e^z - e^z) \\ &= z^3 e^z - 3z^2 e^z + 6z e^z - 6e^z \end{aligned}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech