

Section 5.5

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Exercise 7

$$\begin{aligned} & \int x\sqrt{1-x^2} \, dx \\ & \text{Let : } u = 1 - x^2 \\ & du = -2x \, dx \\ & \int x\sqrt{u} \frac{du}{-2x} \\ & -\frac{1}{2} \int \sqrt{u} \, du \\ & -\frac{1}{2} \left(\frac{2}{3} u^{\frac{3}{2}} \right) \\ & = -\frac{1}{3} (1-x^2)^{\frac{3}{2}} + C \end{aligned}$$

Exercise 15

$$\begin{aligned} & \int \cos^3(\theta) \sin(\theta) \, d\theta \\ & \text{Let : } u = \cos(\theta) \\ & du = -\sin(\theta) \, d\theta \\ & \int u^3 \sin(\theta) \frac{du}{-\sin(\theta)} \\ & - \int u^3 \, du \\ & -\left(\frac{u^4}{4}\right) \\ & = -\frac{\cos^4 \theta}{4} + C \end{aligned}$$

Exercise 21

$$\begin{aligned} & \int \frac{(\ln x)^3}{x} dx \\ & \text{Let : } u = \ln x \\ & du = \frac{1}{x} dx \\ & \int \frac{u^3}{x} dx \\ & \int u^3 dx \\ & \left(\frac{u^4}{4}\right) \\ & = \frac{(\ln x)^4}{4} + C \end{aligned}$$

Exercise 25

$$\begin{aligned} & \int e^x \sqrt{1 + e^x} dx \\ & \text{Let : } u = 1 + e^x \\ & du = e^x dx \\ & \int e^x \sqrt{u} \frac{du}{e^x} \\ & \int \sqrt{u} du \\ & \frac{2}{3} u^{\frac{3}{2}} \\ & = \frac{2}{3} (1 + e^x)^{\frac{3}{2}} + C \end{aligned}$$

Exercise 45

$$\begin{aligned} & \int \frac{1+x}{1+x^2} dx \\ & \text{Let : } u = 1 + x^2 \\ & du = 2x dx \\ & \int \frac{1+x}{1+x^2} = \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx \\ & \int \frac{1}{1+x^2} dx + \int \frac{x}{u} \frac{du}{2x} \end{aligned}$$

$$\int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{1}{u} du$$

$$\tan^{-1}(x) + \frac{1}{2} \ln u$$

$$\tan^{-1}(x) + \frac{1}{2} \ln(1+x^2) + C$$

Exercise 67

$$\int_1^2 x\sqrt{x-1} dx$$

$$\text{Let : } u = x - 1$$

$$du = dx$$

The integral from 1 to 2 becomes 0 to 1 in terms of u .

$$\int_0^1 (u+1)\sqrt{u} du$$

$$\int_0^1 u^{\frac{3}{2}} + u^{\frac{1}{2}} du$$

$$\left[\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} \right]_0^1$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{16}{15}$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech