Alternating Series

Alvin Lin

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Alternating Series

$$\sum_{n=1}^{\infty}(-1)^n b_n = b_1 - b_2 + b_3 - b_4 ...$$

If the following is true:

1. $b_{n+1} \leq b_n \quad \forall \quad n$
2. $\lim_{n\to\infty} b_n \to 0$

Then:

$$\sum_{n=1}^{\infty}(-1)^n b_n \quad \text{converges}$$

Example 1

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n+1}$$

$$b_n = \frac{1}{2n+1}$$

$$b_{n+1} \leq b_n \quad \forall \quad n$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{1}{2n+1} = 0$$

Therefore:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n+1} \quad \text{converges}$$

Example 2

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$$

$$b_n = \frac{1}{\ln(n+4)}$$

$$b_{n+1} \leq b_n \quad \forall \quad n$$

$$\lim_{n\to\infty} \frac{1}{\ln(n+4)} = 0$$

Therefore:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)} \quad \text{converges}$$

Example 3

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3n-1}{2n+1}$$

$$b_n = \frac{3n-1}{2n+1}$$

$$\lim_{n\to\infty} b_n = \frac{3}{2} \neq 0$$

Therefore:

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3n-1}{2n+1} \quad \text{diverges}$$

Practice Problem 11

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{n^3+4}$$

$$b_n = \frac{n^2}{n^3+4}$$

$$b_{n+1} \leq b_n \quad \forall \quad n$$

$$\lim_{n\to\infty} b_n = 0$$

Therefore:

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{n^3+4} \quad \text{converges}$$

Left Comparison Test

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

$$0 < c < \infty$$

Therefore, both sequences either converge or diverge.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech