

# Average Value of a Function

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## Average Value of a Function

Recall the notion of averaging. Given:

$$x_1, x_2, x_3$$
$$x_{average} = \frac{x_1 + x_2 + x_3}{3}$$

The average value of a function from  $x = a$  to  $x = b$  is:

$$f_{av} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

This is a continuous version of the standard average formula, which uses discrete points instead.

## The Mean Value Theorem of Integrals

If  $f$  is continuous on  $[a, b]$ , then there exists  $c$  in  $[a, b]$  such that

$$f(c) = f_{av} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

Therefore:

$$\int_a^b f(x) \, dx = (b-a)f(c)$$

## Practice Problem 9

$$f(x) = (x - 3)^2 \quad [2, 5]$$

$$f_{av} = \frac{1}{b - a} \int_a^b f(x) \, dx$$

$$\frac{1}{5 - 2} \int_2^5 (x - 3)^2 \, dx$$

$$\frac{1}{3} \left[ \frac{(x - 3)^3}{3} \right]_2^5$$

$$\frac{1}{9} \left[ (x - 3)^3 \right]_2^5$$

$$= \frac{1}{9} (8 + 1) = 1$$

$$f(c) = f_{av} = 1$$

$$(c - 3)^3 = 1$$

$$c^2 - 6c + 9 = 1$$

$$c^2 - 6c + 8 = 1$$

$$(c - 2)(c - 4) = 1$$

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)