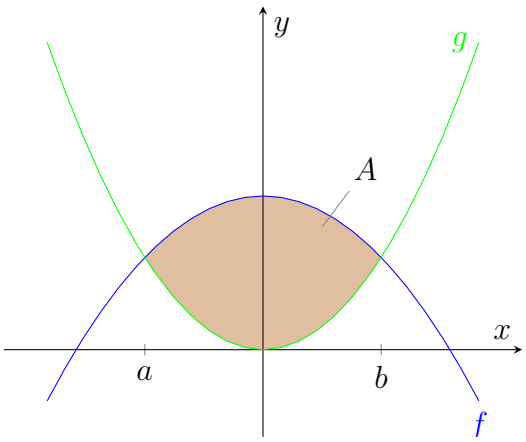
Areas Between Curves

Alvin Lin

Calculus II: August 2016 - December 2016

Areas Between Curves



Finding the area between curves is very similar to finding the area underneath a curve. The area between f and g is simply the difference between the area under f and the area under g.

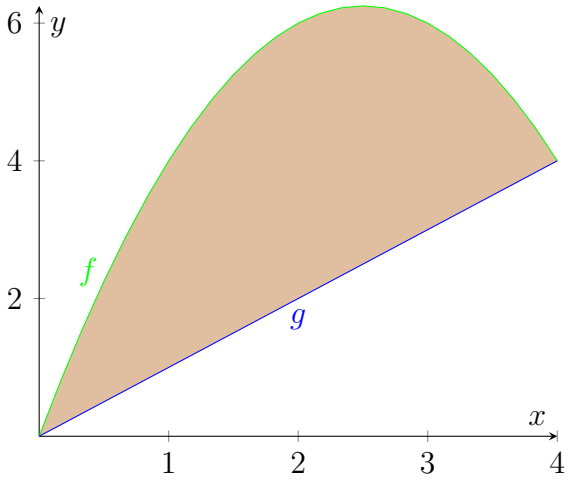
$$A = \int_a^b \Big[f(x) - g(x) \Big] \mathrm{d}x$$

Practice Problem 1

Find the area between the curves:

$$y = 5x - x^2$$

$$y = x$$



We need to find the points of intersection between the curves:

$$x = 5x - x^2$$

$$x^2 - 4 = 0$$

$$x(x-4) = 0$$

$$x = 0 \quad x = 4$$

Now we can use those as the limits of integration:

$$A = \int_a^b \Big[f(x) - g(x) \Big] \mathrm{d}x$$

$$A = \int_0^4 \Big[5x - x^2 - x \Big] \mathrm{d}x$$

$$\int_0^4 \Big[4x - x^2 \Big] \mathrm{d}x$$

$$\left[\frac{4x^2}{2} - \frac{x^3}{3} \right]_0^4$$

$$\frac{4(4^2)}{2} - \frac{4^3}{3} - (0-0)$$

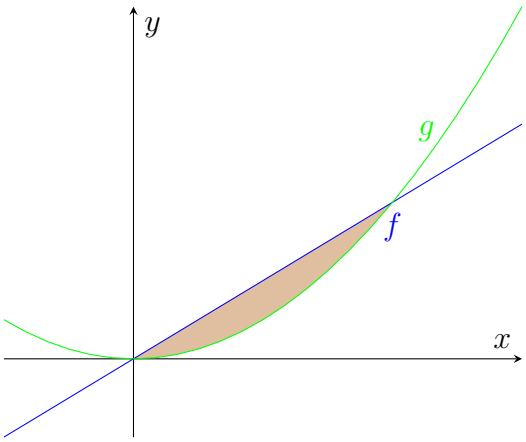
$$= 32 - \frac{64}{3} = \frac{32}{3}$$

Practice Problem 2

Find the area between the curves:

$$y = x$$

$$y = x^2$$



We need to find the points of intersection between the curves:

$$x = x^2$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0 \quad x = 1$$

Now we can use these as the limits of integration:

$$A = \int_0^1 x - x^2 \mathrm{d}x$$

$$\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$\frac{1}{2} - \frac{1}{3} - (0 - 0)$$

$$= \frac{1}{6}$$

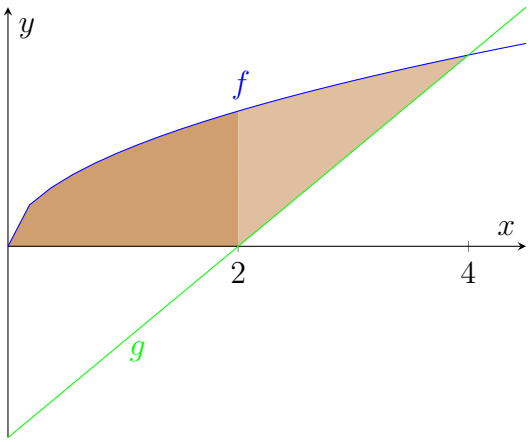
Practice Problem 7

Find the area between the curves in the first quadrant:

$$y = \sqrt{x}$$

$$y = x - 2$$

This problem is trickier, we must split it into two integrals.



$$A = \int_0^2 \sqrt{x}\mathrm{d}x + \int_2^4 \sqrt{x} - (x-2)\mathrm{d}x$$

The first integral is just the area under the curve of $\sqrt{x}$ from 0 to 2, while the second integral is the area between the curves from 2 to 4.

$$\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2 + \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2 + 2x\right]_2^4$$

$$\frac{2}{3}(2^{\frac{3}{2}}) + \left[\frac{2}{3}4^{\frac{3}{2}} - \frac{1}{2}4^2 + 2(4)\right] - \left[\frac{2}{3}2^{\frac{3}{2}} - \frac{1}{2}2^2 + 2(2)\right]$$

$$\frac{2}{3}(2^{\frac{3}{2}}) + \frac{2}{3}4^{\frac{3}{2}} - \frac{1}{2}4^2 + 2(4) - \frac{2}{3}2^{\frac{3}{2}} + \frac{1}{2}2^2 - 2(2)$$

$$\frac{16}{3} - 8 + 8 + 4 - 4$$

$$= \frac{16}{3}$$

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech