Introduction to Cryptography

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Block Ciphers

A stream cipher algorithm defines how to encrypt or decrypt arbitrary length messages. A block cipher can only encrypt or decrypt b bits, no more, no less. Arbitrary length message are handled by a separate algorithm called a block cipher mode of operation (ECB). A stream cipher's encryption and decryption operations are the same. A block cipher's encryption and decryption operations are different. To decrypt, the encryption operation must be run backward. Thus, the encryption operation must be invertible. Additionally, block ciphers are much more than just an encryption algorithm, they can be used:

- to build different types of block-based encryption schemes
- to realize stream ciphers
- to construct hash functions
- to make message authentication codes
- to build key establishment protocols
- to make a pseudo-random number generator

Block Cipher Primitives

Claude Shannon: there are two primitive operations with which strong encryption algorithms can be built.

1. **Confusion:** An encryption operation where the relationship between the key and ciphertext is obscured. Today, a common element for achieving confusion is substitution, which is found in both AES and DES.

2. **Diffusion:** An encryption operation where the influence of one plaintext symbol is spread of over many ciphertext symbols with the goal of hiding statistical properties of the plaintext. A simple diffusion element is the bit permutation, which is frequently used within DES. This obscures the relationship between the ciphertext and the plaintext.

Both operations by themselves cannot provide security. The idea is to concatenate confusion and diffusion elements to build so called product ciphers. Most of today's block ciphers are product ciphers as they consist of rounds which are applied repeatedly to the data. This can reach excellent diffusion, as changing one bit of the plaintext results on average in the change of half the output bits.

Block Cipher Modes of Operation

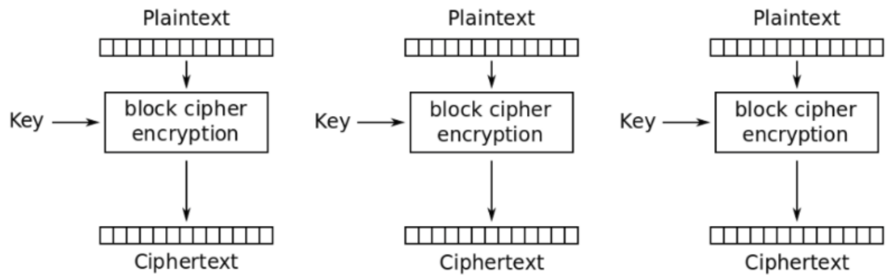
There are several ways of encrypting long plaintexts with a block cipher:

- Electronic Code Book mode (ECB)
- Cipher Block Chaining mode (CBC)
- Output Feedback mode (OFB)
- Cipher Feedback mode (CFB)
- Counter mode (CTR)
- Galois Counter Mode (GCM)

These modes have the goal of providing authenticity and integrity in addition to confidentiality. It ascertains that the message is coming from the original sender and that it was not altered during transmission.

Electronic Code Book mode (ECB)

This is the simplest of the encryption modes. Messages which exceed b bits are partitioned into b-bit blocks, which are encrypted separately. In case the plaintext message's length is not a multiple of b, we add padding to the end.



Advantages:

- No block synchronization between the sender and receiver is required.
- Bit errors caused by noisy channels only affect the corresponding block but not succeeding blocks.
- Encryption can be parallelized, which is an advantage for high-speed implementations.

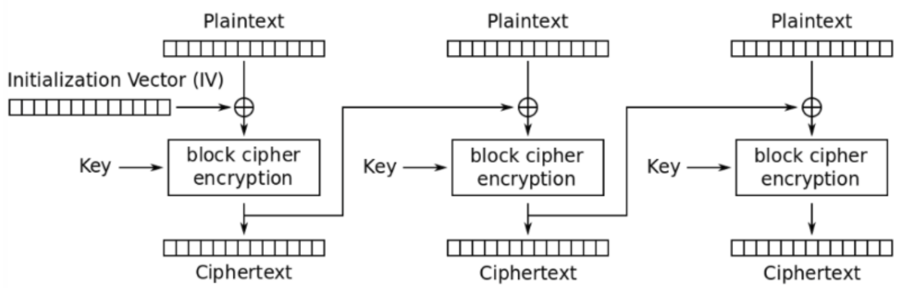
Disadvantages:

- Encryption is highly deterministic.
- Identical plaintexts result in identical ciphertexts.
- An attacker recognizes if the same message has been sent twice.
- Plaintext blocks are encrypted independently of previous blocks.
- An attacker may reorder ciphertext blocks which result in valid plaintext.
- Statistical properties in the plaintext are preserved in the ciphertext.

Once a particular plaintext to ciphertext block mapping is known, a sequence of ciphertext blocks can easily be manipulated.

Cipher Block Chaining mode (CBC)

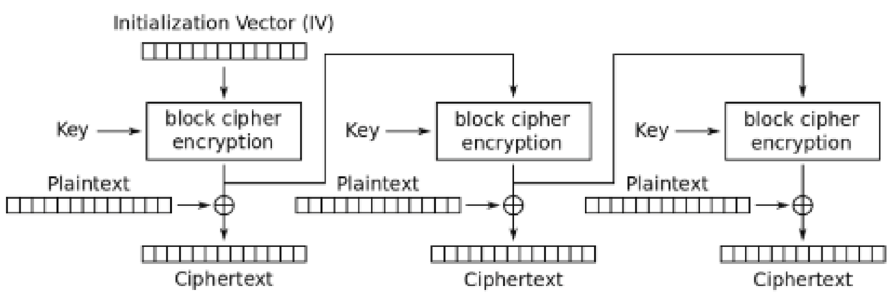
There are two main ideas behind this mode: the encryption of all blocks are chained together, and the encryption is randomized by using an initialization vector.



This mode also requires padding, but messages longer than $2^{\frac{n}{2}}$ blocks (where n is the block size in bits) shouldn't be encrypted with this mode, since it gives an attacker more information about the ciphertext. Encryption cannot be parallelized, but decryption can be. The initialization vector does not need to be secret, but it should be a randomized nonce.

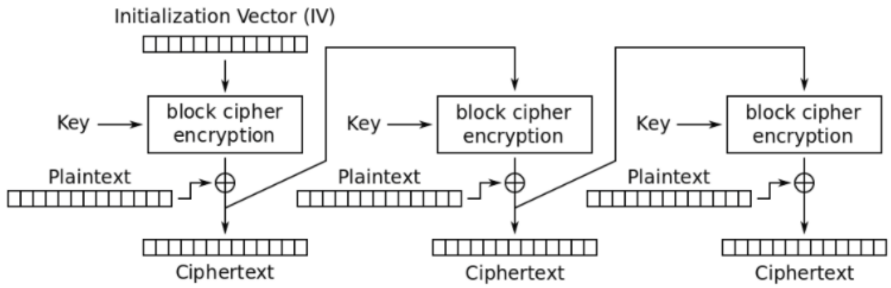
Output Feedback mode (OFB)

This mode builds a synchronous stream cipher from a block cipher, where the key stream is generated in a blockwise fashion. The output of the cipher gives us key stream bits with which we can encrypt plaintext bits using the XOR operation. This mode also requires an initialization vector (which also should be a randomized nonce).



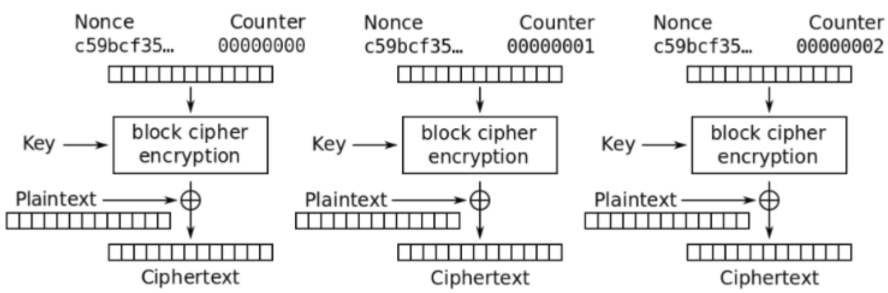
Cipher Feedback mode (CFB)

This mode uses a block cipher as a building block for an synchronous stream cipher. The key stream is generated in a blockwise fashion and is also a function of the ciphertext. Since it also uses a randomized initialization vector, the ciphertext is also nondeterministic. It can be used in situations where short plaintext blocks are to be encrypted, but has no real advantage over Output Feedback mode.



Counter mode (CTR)

This mode turns a block cipher into a stream cipher where the keystream is computed in a blockwise fashion (similar to OFB and CFB mode). The input to the block cipher is a counter which assumes a different value every time the block cipher computes a new key stream block. Unlike CFB, and OFB however, this is highly parallelizable and does not require padding since the unneeded portion of the last key block can be discarded (because it is a stream cipher).



Feistel Cipher (Feistel network)

Horst Feistel proposed the use of a cipher that alternates substitutions and permutations. This is a practical application of a proposal by Claude Shannon to develop a product cipher. A large set of block ciphers use the scheme (it is a design model from which many different block ciphers are derived), including the Data Encryption Standard (DES). DES is just one example of a Feistel Cipher.

Data Encryption Standard (DES)

- Data Encryption Standard (DES) encrypts blocks of size 64 bit.

- Developed by IBM based on the cipher Lucifer under the influence of the National Security Agency (NSA).
- Standardized in 1977 by the National Bureau of Standards (NBS), now called the National Institute of Standards and Technology (NIST).
- Most popular block cipher for most of the last 30 years.
- By far the best studied symmetric algorithm.
- Nowadays considered insecure due to the small key length of 56 bits.
- Replaced by the Advanced Encryption Standard (AES) in 2000.

By encrypting with DES three times in a row, triple DES (3DES) was created, against which no practical attack is currently known. 3DES is still widely in use today.

DES Algorithm Internals

The DES algorithm is a symmetric cipher that encrypts 64-bit blocks using a 56-bit key. It performs 16 rounds of operations on the plaintext, performing the following sequence of operations 16 times:

1. **Expansion function E:** increases diffusion by taking the 32 input bits and duplicating some to fit them into a 48 bit block.

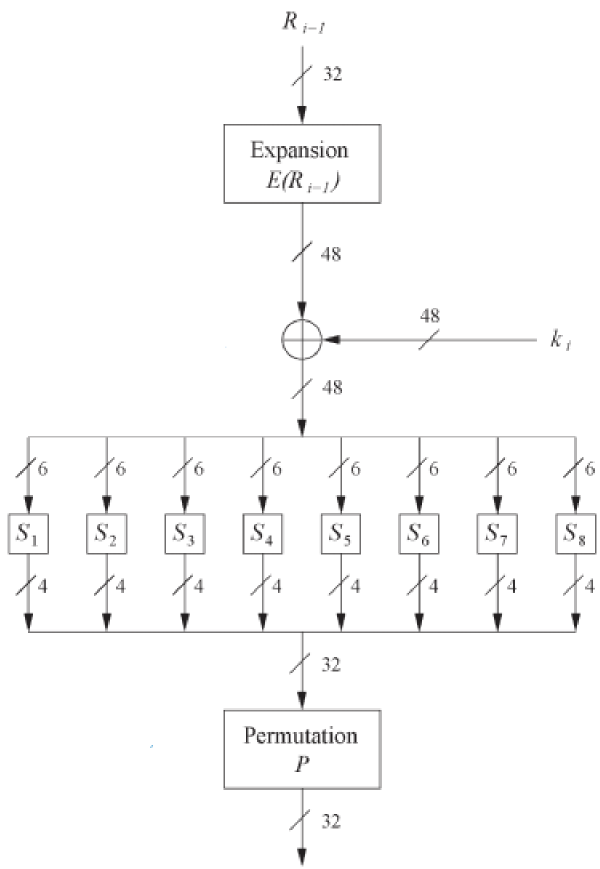
2. **XOR with round key:** the 48-bit block is XOR'ed with a 48-bit round subkey. A different subkey derived from the main key is used in each round.

3. **Substitution using S-boxes:** the 48-bit block is split into 8 chunks that are 6 bits long and a predetermined 4×16 substitution box is used to translate the 6-bit input into a 4-bit output. The first and last bit determine the row of the substitution box to use while the middle four bits determine the column of the substitution box to use. This is the S-box used in round one of the DES algorithm:

S_1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	14	04	13	01	02	15	11	08	03	10	06	12	05	09	00	07
1	00	15	07	04	14	02	13	01	10	06	12	11	09	05	03	08
2	04	01	14	08	13	06	02	11	15	12	09	07	03	10	05	00
3	15	12	08	02	04	09	01	07	05	11	03	14	10	00	06	13

The bit sequence 100101 would be substituted with the entry in row 11 (4 in decimal) and column 0010 (3 in decimal). Using the S-box above, this would be the value 08 (1000 in binary). The S-boxes are crucial for DES security since they introduce a non-linear element, making this algorithm resistant to differential analysis.

4. **Permutation:** a bitwise permutation is performed to introduce diffusion. The sum of all these operations ensures that after round 5, every bit of output is a function of each key bit and each plaintext bit.



Because DES is a Feistel Cipher, only the key schedule needs to be modified for decryption. The round subkeys need to be generated in reverse order, and after that the encryption algorithm can be applied using the keys in reverse to decrypt the data.

Analytical Attacks

DES is quite robust against known analytical attacks such as linear and differential analysis. In practice, it is very difficult to break the cipher with linear or analytical cryptanalysis.

Exhaustive Key Search

A simple exhaustive search for a DES key pair knowing one plaintext-ciphertext pair is relatively easy given today's computer technology since DES has a key space of size 2^{56}. However, for most other block ciphers, a key search is somewhat more complicated. A brute-force attack can produce false positive results, where a key is found that is not the one used for the encryption. The likelihood of this is related to the relative size of the key space and the plaintext space.

Assume a cipher with a block width of 64 bits and a key size of 80 bits is used to encrypt a plaintext. If we encrypt once under all possible 2^{80} keys, we obtain 2^{80} possible ciphertexts. However, there exist only 2^{64} different ones. If we run through all keys for a given plaintext-ciphertext pairs, we find on average $\frac{2^{80}}{2^{64}} = 2^{16}$ keys that perform the mapping $e_k(x) = y$. Given a block cipher with a key length of k bits and a block size of n bits, as well as t plaintext-ciphertext pairs, the expected number of false keys which encrypts all plaintexts to the corresponding ciphertexts is 2^{k-tn}.

Advanced Encryption Standard (AES)

- AES is the most widely used symmetric cipher today
- The algorithm for AES was chosen by the US National Institute of Standards and Technology (NIST) in a multi-year selection process, requiring all AES candidate submissions to be block ciphers with 128-bit block sizes supporting 128-bit, 192-bit, and 256-bit keys.
- 5 finalists were announced in August 1999:
  - *Mars* - IBM Corporation
  - *RC6* - RSA Laboratories
  - *Rijndael* - J. Daemen and V. Rijmen
  - *Serpent* - Eli Biham et al.
  - *Twofish* - B. Schneier et al.

Rijndael was chosen as the new AES in October 2000 and formally approved as a US federal standard in November 2001.

- Like DES, AES operates by applying rounds of operations to the plaintext. The number of rounds performed depends on the key length.

Key length (bits)	Number of rounds
128	10
192	12
256	14

AES Algorithm Internals

The AES algorithm is a byte-oriented cipher, so the 128-bit cipher text is arranged into a 4×4 matrix of bytes. Each round consists of layers, which perform the following operations on the matrix:

1. **Byte Substitution Layer:** 16 identical S-boxes are used to map each byte in the matrix to some new value. These S-boxes are the only nonlinear elements of AES and are bijective, so they can be uniquely reversed.

2. **Diffusion Layer:** the input state bits are passed through two sub-layers:

   - **ShiftRows Sublayer:** a bytewise permutation is performed, which left shifts the first row of the matrix once, left shifts the second row twice, and left shifts the third row three times.
   - **MixColumn Sublayer:** each 4-byte column of the matrix is multiplied by some fixed 4×4 matrix to mix each column of the state matrix.

   This layer provides diffusion over all the input state bits.

3. **Key Addition Layer:** the 16-byte state matrix is XOR'ed with a 16-byte subkey generated in the key schedule, derived from the original input key.

This process is reversed for decryption. Since AES is not based on a Feistel network, all the layers must be reversed for decryption. Generally, this means that all the subkeys must be conputed before the decryption of the first block can begin.

Implementation in Software

NIST also required that AES had the possibility of an efficient software implementation. A straightforward implementation is well suited for 8-bit processors, but inefficient on 32-bit and 64-bit processors. A more sophisticated approach would be to merge all the layer functions except the key addition into a table lookup, allowing a round to be computed using 16 table lookups.

Security

Due to the key length, a brute force attack is infeasible. There exist no known analytical attacks, but several side-channel attacks have been published. These attacks target implementations of the algorithm however, rather than weaknesses in the underlying algorithm.

You can find all my notes at `http://omgimanerd.tech/notes`. If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech