

# Intro to Computer Science Theory: Homework 4

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Let  $k \in \mathbb{N}$  be an arbitrary natural number. Show that the following languages over the alphabet  $\{0, 1\}$  are regular.

1.  $L_1 = \{x \mid |x| \geq k\}$  is accepted by the finite automaton  $M = \{Q, \Sigma, \delta, q_0, F\}$  such that:

- $Q = \{n \mid (1 \leq n \leq k \wedge n \in \mathbb{N})\}$
- $\Sigma = \{0, 1\}$
- $\delta: Q \times \Sigma \rightarrow Q$  is defined on  $(q, z) \in Q \times \Sigma$  as:

$$\delta(q, x) = \begin{cases} q & \text{if } q = k \\ q + 1 & \text{otherwise} \end{cases}$$

- $q_0 = 1$
- $F = \{k\}$

Therefore,  $L_1$  is a regular language.

2.  $L_2 = \{x \mid |x| \equiv 1 \pmod{k}\}$  is accepted by the finite automaton  $M = \{Q, \Sigma, \delta, q_0, F\}$  such that:

- $Q = \{n \mid (1 \leq n \leq k \wedge n \in \mathbb{N})\}$
- $\Sigma = \{0, 1\}$
- $\delta: Q \times \Sigma \rightarrow Q$  is defined on  $(q, z) \in Q \times \Sigma$  as:

$$\delta(q, x) = (q + 1) \pmod{k}$$

- $q_0 = 1$
- $F = \{2\}$

Therefore,  $L_2$  is a regular language.

3.  $L_3 = \{x \mid x, \text{ interpreted as a binary number, is at least } k\}$  is accepted by the finite automaton  $M = \{Q, \Sigma, \delta, q_0, F\}$  such that:

- $Q = \{n \mid (0 \leq n \leq k \wedge n \in \mathbb{N})\}$
- $\Sigma = \{0, 1\}$
- $\delta: Q \times \Sigma \rightarrow Q$  is defined on  $(q, z) \in Q \times \Sigma$  as:

$$\delta(q, x) = \begin{cases} k & \text{if } (q = k) \vee (2q + x \geq k) \\ 2q + x & \text{otherwise} \end{cases}$$

- $q_0 = 0$
- $F = \{k\}$

Therefore,  $L_3$  is a regular language.

4.  $L_4 = \{x \mid x, \text{ interpreted as a binary number, is divisible by } k\}$  is accepted by the finite automaton  $M = \{Q, \Sigma, \delta, q_0, F\}$  such that:

- $Q = \{n \mid (0 \leq n < k \wedge n \in \mathbb{N})\}$
- $\Sigma = \{0, 1\}$
- $\delta: Q \times \Sigma \rightarrow Q$  is defined on  $(q, z) \in Q \times \Sigma$  as:

$$\delta(q, x) = (2q + x) \pmod k$$

- $q_0 = 0$
- $F = \{0\}$

Therefore,  $L_4$  is a regular language.

If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)