

Intro to Computer Science Theory: Homework 4

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Let $k \in \mathbb{N}$ be an arbitrary natural number. Show that the following languages over the alphabet $\{0, 1\}$ are regular.

1. $L_1 = \{x \mid |x| \geq k\}$ is accepted by the finite automaton $M = \{Q, \Sigma, \delta, q_0, F\}$ such that:

- $Q = \{n \mid (1 \leq n \leq k \wedge n \in \mathbb{N})\}$
- $\Sigma = \{0, 1\}$
- $\delta: Q \times \Sigma \rightarrow Q$ is defined on $(q, z) \in Q \times \Sigma$ as:

$$\delta(q, x) = \begin{cases} q & \text{if } q = k \\ q + 1 & \text{otherwise} \end{cases}$$

- $q_0 = 1$
- $F = \{k\}$

Therefore, L_1 is a regular language.

2. $L_2 = \{x \mid |x| \equiv 1 \pmod{k}\}$ is accepted by the finite automaton $M = \{Q, \Sigma, \delta, q_0, F\}$ such that:

- $Q = \{n \mid (1 \leq n \leq k \wedge n \in \mathbb{N})\}$
- $\Sigma = \{0, 1\}$
- $\delta: Q \times \Sigma \rightarrow Q$ is defined on $(q, z) \in Q \times \Sigma$ as:

$$\delta(q, x) = (q + 1) \pmod{k}$$

- $q_0 = 1$
- $F = \{2\}$

Therefore, L_2 is a regular language.

3. $L_3 = \{x \mid x, \text{ interpreted as a binary number, is at least } k\}$ is accepted by the finite automaton $M = \{Q, \Sigma, \delta, q_0, F\}$ such that:

- $Q = \{n \mid (0 \leq n \leq k \wedge n \in \mathbb{N})\}$
- $\Sigma = \{0, 1\}$
- $\delta: Q \times \Sigma \rightarrow Q$ is defined on $(q, z) \in Q \times \Sigma$ as:

$$\delta(q, x) = \begin{cases} k & \text{if } (q = k) \vee (2q + x \geq k) \\ 2q + x & \text{otherwise} \end{cases}$$

- $q_0 = 0$
- $F = \{k\}$

Therefore, L_3 is a regular language.

4. $L_4 = \{x \mid x, \text{ interpreted as a binary number, is divisible by } k\}$ is accepted by the finite automaton $M = \{Q, \Sigma, \delta, q_0, F\}$ such that:

- $Q = \{n \mid (0 \leq n < k \wedge n \in \mathbb{N})\}$
- $\Sigma = \{0, 1\}$
- $\delta: Q \times \Sigma \rightarrow Q$ is defined on $(q, z) \in Q \times \Sigma$ as:

$$\delta(q, x) = (2q + x) \pmod k$$

- $q_0 = 0$
- $F = \{0\}$

Therefore, L_4 is a regular language.

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech