Intro to Computer Science Theory: Homework 2

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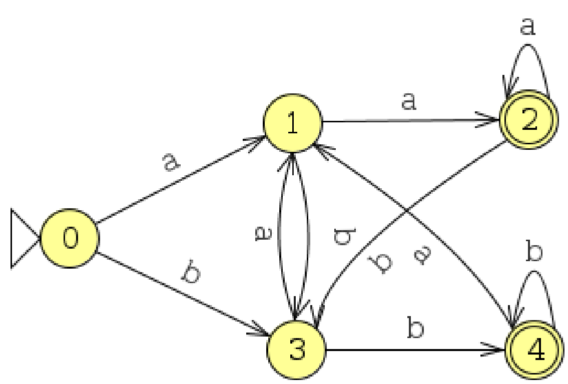
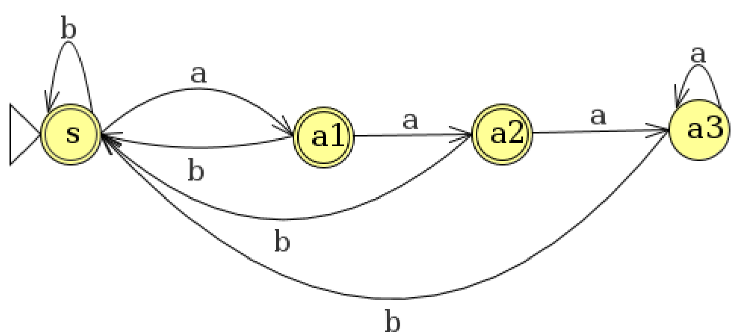
Problem 1

Give FA state transition diagrams for the following languages. Be sure to label the names of your states. Each language is over the alphabet $\{a, b\}$, and is defined as the set of all strings.

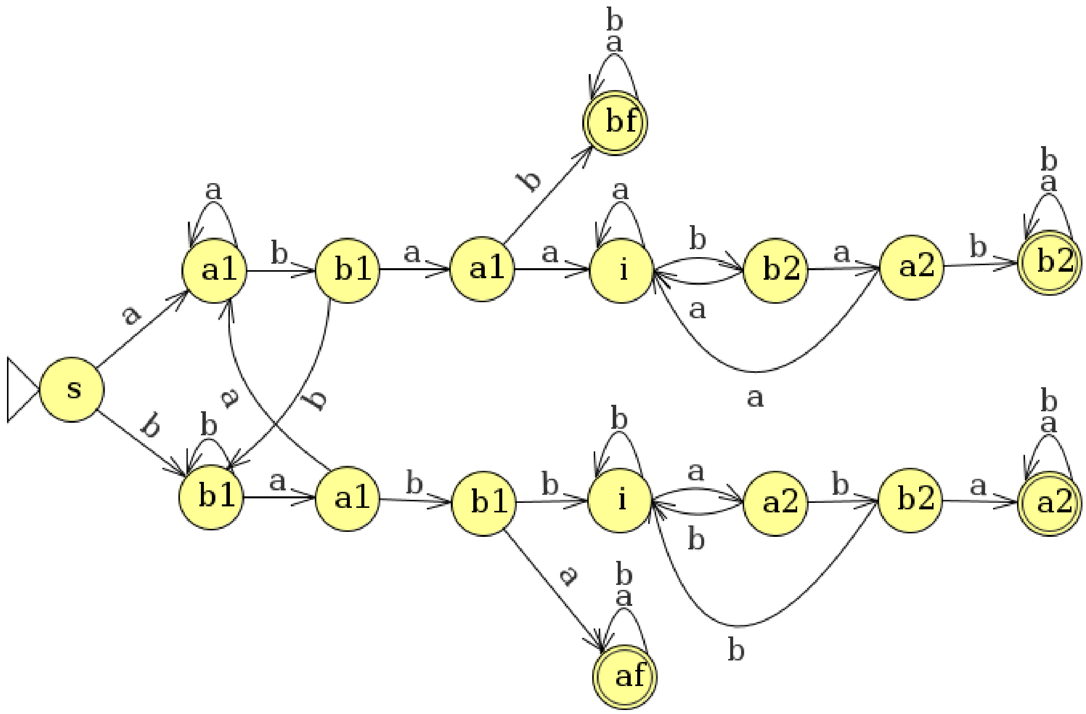
1. that begin or end in aa in bb.



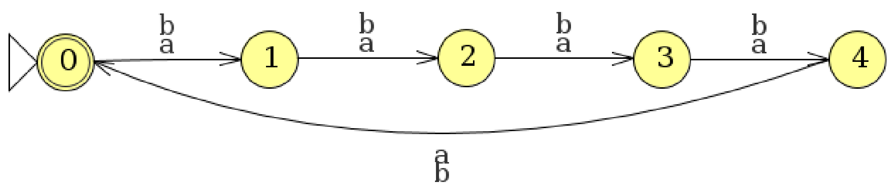
2. that do not have aaa as a substring.



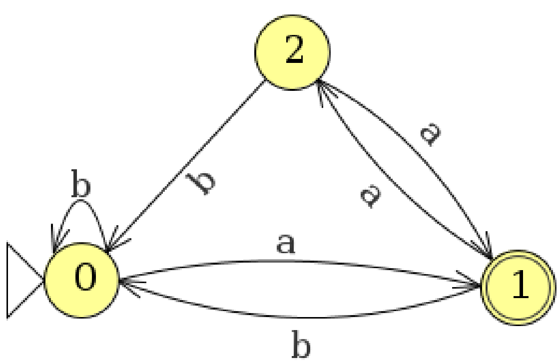
3. that contain both aba and bab as substrings.



4. whose length is a multiple of 5.



5. where the number of a's after the last b in the string is odd.



Problem 2

Provide formal definitions for 1, 4, and 5 above. Try to make them as concise as possible.

Finite Automaton 1

$M = \{Q, \Sigma, \delta, 0, \{2, 4\}\}$ where:

- $Q = \{0, 1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- $\delta : Q \times E \to Q$ is defined on $(q, z) \in Q \times \Sigma$ as:

$$\delta(q, x) = \begin{cases} 1 & if\ (x = a) \wedge (q \in \{0, 3, 4\}) \\ 2 & if\ (x = a) \wedge (q \in \{1, 2\}) \\ 3 & if\ (x = b) \wedge (q \in \{0, 1, 2\}) \\ 4 & if\ (x = b) \wedge (q \in \{3, 4\}) \end{cases}$$

Finite Automaton 4

$M = \{Q, \Sigma, \delta, 0, \{0\}\}$ where:

- $Q = \{0, 1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- $\delta : Q \times E \to Q$ is defined on $(q, z) \in Q \times \Sigma$ as:

$$\delta(q, x) = q + 1$$

Finite Automaton 5

$M = \{Q, \Sigma, \delta, 0, \{1\}\}$ where:

- $Q = \{0, 1, 2\}$
- $\Sigma = \{a, b\}$
- $\delta : Q \times E \to Q$ is defined on $(q, z) \in Q \times \Sigma$ as:

$$\delta(q, x) = \begin{cases} 0 & if\ (x = b) \\ q + 1 & if\ (x = a) \wedge (q \neq 2) \\ 1 & if\ (x = a) \wedge (q = 2) \end{cases}$$

Problem 3

Prove, in the following steps, that for any language L, if $L \circ L \subseteq L$ then $L = \{\epsilon\}$ or L is infinite.

1. Write the theorem using quantifiable logic, as in our previous homework.

$$(L \circ L \subseteq L) \to ((L = \{\epsilon\}) \vee L \text{ is infinite})$$

2. Rewrite the following statement in quantifiable logic. For all natural numbers n, there exists an x in L such that $|x| > n$. Note that this is effectively what it means for L to be infinite.

$$\forall n(n \in \mathbb{N}) \exists x(x \in L \wedge |x| > n)$$

3. Where appropriate, substitute into your answer in 1 for the definitions of $\circ, \subseteq$, and "L is infinite".

$$\forall x((x \in L \circ L) \to (x \in L)) \to ((L = \{\epsilon\}) \vee (\forall n(n \in \mathbb{N}) \exists x(x \in L \wedge |x| > n)))$$

4. Write the following statements in predicate logic.

   (a) For any string x, $|x| = 0$ if and only if $x = \epsilon$.

   $$\forall x(|x| = 0) \leftrightarrow (x = \epsilon)$$

   (b) For any strings x and y, $|xy| = |x| + |y|$.

   $$\forall x \forall y(|xy| = |x| + |y|)$$

If you have any questions, comments, or concerns, please contact me at alvin@omgimanerd.tech