

# Intro to Computer Science Theory: Homework 2

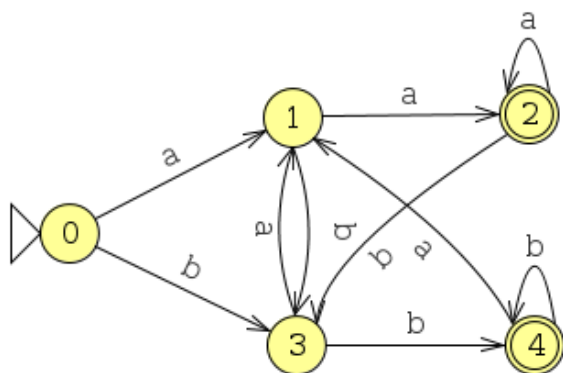
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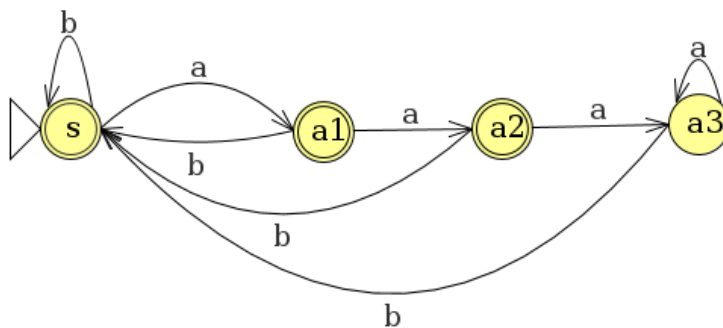
## Problem 1

Give FA state transition diagrams for the following languages. Be sure to label the names of your states. Each language is over the alphabet  $\{a, b\}$ , and is defined as the set of all strings.

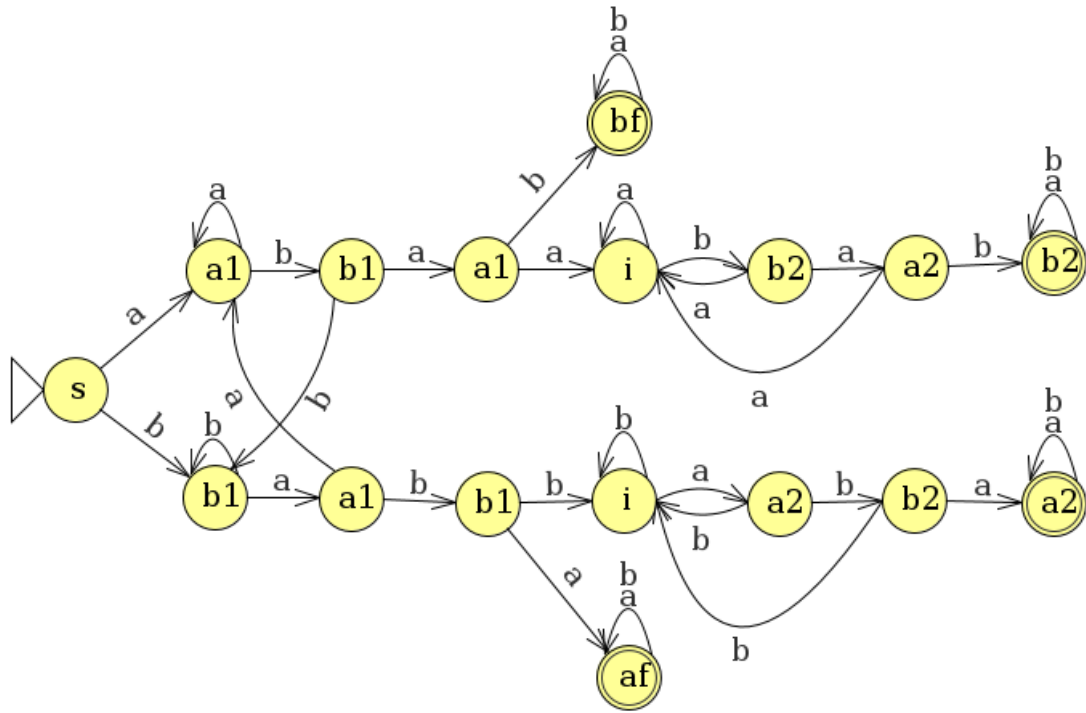
1. that begin or end in  $aa$  in  $bb$ .



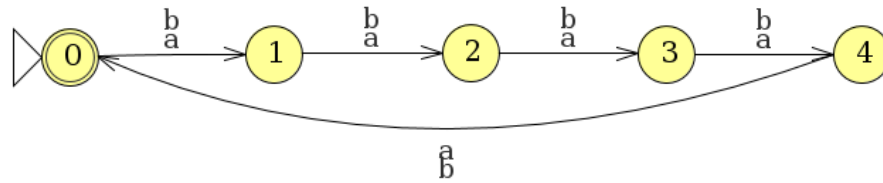
2. that do not have  $aaa$  as a substring.



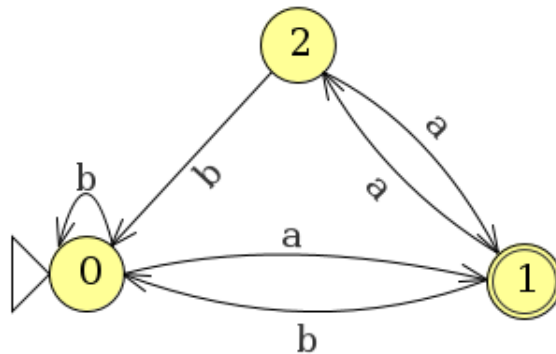
3. that contain both  $aba$  and  $bab$  as substrings.



4. whose length is a multiple of 5.



5. where the number of *a*'s after the last *b* in the string is odd.



## Problem 2

Provide formal definitions for 1, 4, and 5 above. Try to make them as concise as possible.

### Finite Automaton 1

$M = \{Q, \Sigma, \delta, 0, \{2, 4\}\}$  where:

- $Q = \{0, 1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- $\delta : Q \times E \rightarrow Q$  is defined on  $(q, z) \in Q \times \Sigma$  as:

$$\delta(q, x) = \begin{cases} 1 & \text{if } (x = a) \wedge (q \in \{0, 3, 4\}) \\ 2 & \text{if } (x = a) \wedge (q \in \{1, 2\}) \\ 3 & \text{if } (x = b) \wedge (q \in \{0, 1, 2\}) \\ 4 & \text{if } (x = b) \wedge (q \in \{3, 4\}) \end{cases}$$

### Finite Automaton 4

$M = \{Q, \Sigma, \delta, 0, \{0\}\}$  where:

- $Q = \{0, 1, 2, 3, 4\}$
- $\Sigma = \{a, b\}$
- $\delta : Q \times E \rightarrow Q$  is defined on  $(q, z) \in Q \times \Sigma$  as:

$$\delta(q, x) = q + 1$$

### Finite Automaton 5

$M = \{Q, \Sigma, \delta, 0, \{1\}\}$  where:

- $Q = \{0, 1, 2\}$
- $\Sigma = \{a, b\}$
- $\delta : Q \times E \rightarrow Q$  is defined on  $(q, z) \in Q \times \Sigma$  as:

$$\delta(q, x) = \begin{cases} 0 & \text{if } (x = b) \\ q + 1 & \text{if } (x = a) \wedge (q \neq 2) \\ 1 & \text{if } (x = a) \wedge (q = 2) \end{cases}$$

## Problem 3

Prove, in the following steps, that for any language  $L$ , if  $L \circ L \subseteq L$  then  $L = \{\epsilon\}$  or  $L$  is infinite.

1. Write the theorem using quantifiable logic, as in our previous homework.

$$(L \circ L \subseteq L) \rightarrow ((L = \{\epsilon\}) \vee L \text{ is infinite})$$

2. Rewrite the following statement in quantifiable logic. For all natural numbers  $n$ , there exists an  $x$  in  $L$  such that  $|x| > n$ . Note that this is effectively what it means for  $L$  to be infinite.

$$\forall n(n \in \mathbb{N}) \exists x(x \in L \wedge |x| > n)$$

3. Where appropriate, substitute into your answer in 1 for the definitions of  $\circ$ ,  $\subseteq$ , and “ $L$  is infinite”.

$$\forall x((x \in L \circ L) \rightarrow (x \in L)) \rightarrow ((L = \{\epsilon\}) \vee (\forall n(n \in \mathbb{N}) \exists x(x \in L \wedge |x| > n)))$$

4. Write the following statements in predicate logic.

(a) For any string  $x$ ,  $|x| = 0$  if and only if  $x = \epsilon$ .

$$\forall x(|x| = 0) \leftrightarrow (x = \epsilon)$$

(b) For any strings  $x$  and  $y$ ,  $|xy| = |x| + |y|$ .

$$\forall x \forall y(|xy| = |x| + |y|)$$

If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)