

# CSCI 251: Concepts of Parallel and Distributed Systems

Alvin Lin

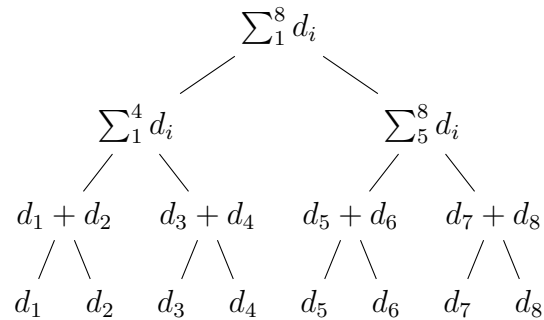
August 30th, 2017

## Topics

- Parallel Addition
- Speed up
- Amdahl's Law
- Efficiency
- Scalability
- Superlinear speedup
- Merge Sort

## Parallel Addition

Suppose we have  $N$  numbers to add. A serial computation would require  $(N-1)$  additions, making it an  $O(N)$  operation. If we were to add  $N$  numbers 2 at a time in a tree structure like manner, then it would only take  $\log_2 N$  steps.



## Speed Up

The speed up is the time taken to execute a given problem on a single computer over the time take to execute the given problem on a parallel computer. We make several assumptions about the parallel computer:

- each processor runs on the same clock speed
- the bandwidth is the same for all communications
- the memory access times are the same

## Message Passing Model

Each of the steps in the tree above requires a communication step as well as the addition operation.

$$\begin{array}{cccccccc}
 d_1 \leftarrow & d_2 & d_3 \leftarrow & d_4 & d_5 \leftarrow & d_6 & d_7 \leftarrow & d_8 \\
 P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8
 \end{array}$$

$$\begin{array}{cccc}
 d_1 + d_2 \leftarrow & d_3 + d_4 & d_5 + d_6 \leftarrow & d_7 + d_8 \\
 \dots & & &
 \end{array}$$

Even numbered processors will pass a message to  $P_{i-1}$ .

## Shared Memory Model

Alternatively, there can exist a region of shared memory which all processors  $P_i$  can access. During the communication step, each processor  $P_i$  writes  $d_i$  into shared memory. The flow will be as follows:

- memory write
- memory read
- add
- memory write

There needs to be a mechanism (mutex lock/semaphore) to make the shared memory operations atomic.

## Modeling the speed up

The speed up is limited by the portion of the program that can be parallelized. Suppose in a given problem,  $\alpha$  is the portion that cannot be parallelized.  $(1 - \alpha)$  is the portion that can be parallelized. Given that the time taken to execute the algorithm on a single computer is  $T_s = 1$ , the time taken to execute the same algorithm on a parallel computer with  $P$  processors is given by:

$$T_p = \frac{1 - \alpha}{P} + \alpha$$

The speed up  $S$  is given by:

$$\begin{aligned} S &= \frac{T_s}{T_p} \\ &= \frac{1}{\frac{1-\alpha}{P} + \alpha} \end{aligned}$$

## Amdahl's Law

This speed is bounded by  $\frac{1}{\alpha}$ , thus you cannot get a speed up greater than  $\frac{1}{\alpha}$ . Suppose  $\alpha = 0.2$ , meaning 20% of the program cannot be parallelized. With 2 processors:

$$S = \frac{1}{\frac{0.8}{2} + 0.2} = \frac{1}{0.6} = \frac{5}{3}$$

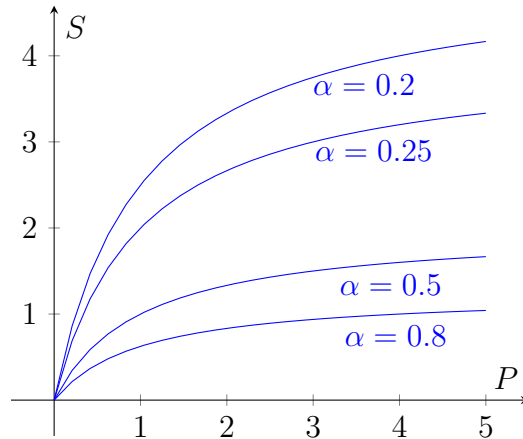
With  $P = 4$ :

$$S = \frac{1}{\frac{0.8}{4} + 0.2} = \frac{1}{0.4} = 2.5$$

With  $P = 8$ :

$$S = \frac{1}{0.3} = 3.33$$

Graphing this, we can see the limitations of the speed up in that it is upper bounded by  $\frac{1}{\alpha}$ :



## Efficiency

The efficiency  $E$  is equal to the speed up  $S$  over the number of processors  $P$ :

$$\begin{aligned} E &= \frac{S}{P} \\ &= \left[ \frac{1}{\frac{1-\alpha}{P} + \alpha} \right] \frac{1}{P} \end{aligned}$$

## Scalability

In the initial addition problem,  $N$  was equal to  $P$ . In real world situations,  $N$  is often far greater than  $P$ . For  $N = 64$  and  $P = 4$ , each processor should get  $\frac{N}{P}$  data

elements.

$$P_1 \rightarrow \sum_1^{16} d_i$$
$$P_2 \rightarrow \sum_{17}^{32} d_i$$
$$P_3 \rightarrow \sum_{33}^{48} d_i$$
$$P_4 \rightarrow \sum_{49}^{64} d_i$$

$\frac{N}{P} - 1$  additions on each processor is  $O(\frac{N}{P})$  with  $\log_2 P$  steps, or  $O(\frac{N}{P} + \log(P))$ .

## Superlinear speedup

Depth-First Search Trees, search time can be halved by dividing the task between two computers.

## Reminders

Professor Mohan Kumar:  
mjkvcs@rit.edu  
<https://cs.rit.edu/~mjk>

Jennifer Burt (Additional Contact):  
jennifer@cs.rit.edu

## Homework

Check MyCourses for sample questions. There will be a small quiz in class next Wednesday.

You can find all my notes at <http://omgimanerd.tech/notes>. If you have any questions, comments, or concerns, please contact me at [alvin@omgimanerd.tech](mailto:alvin@omgimanerd.tech)